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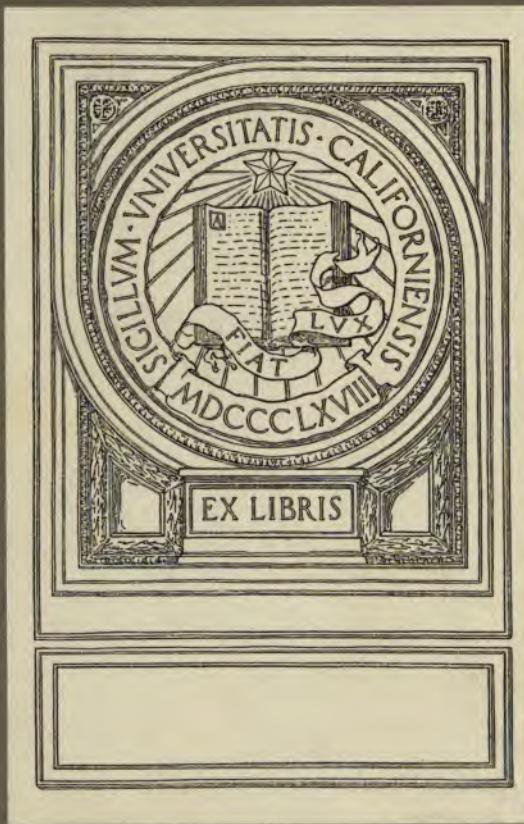
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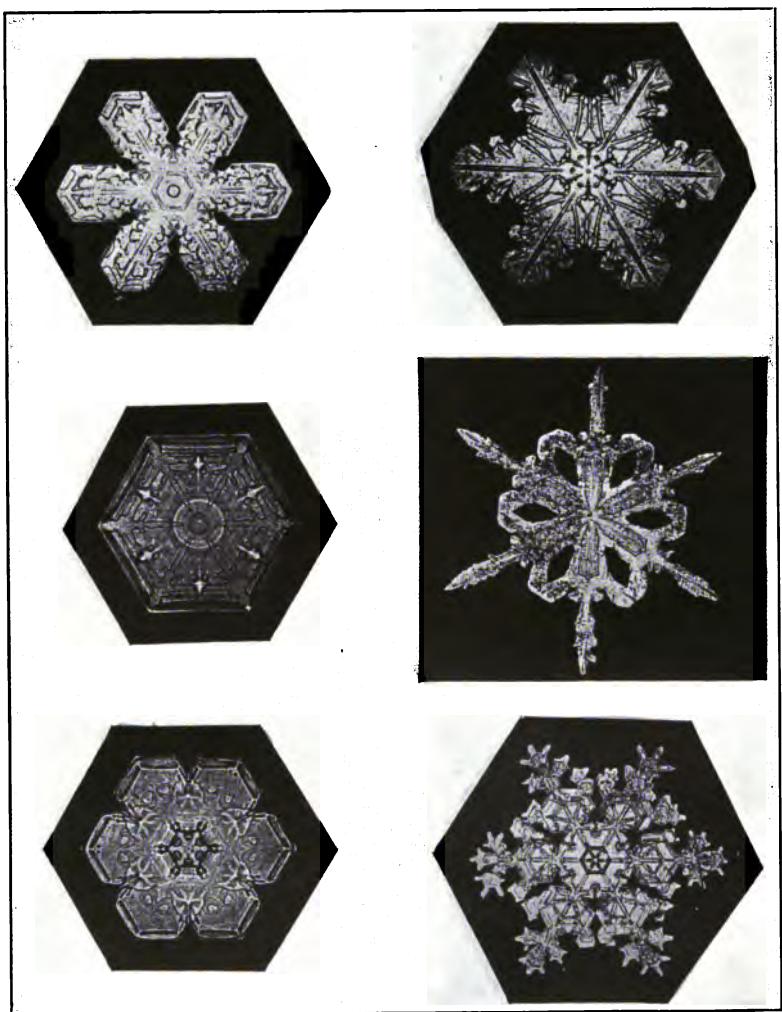
# CRYSTALLOGRAPHY

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*(After Chamberlin and Salisbury)*

**Fig. 1**

# CRYSTALLOGRAPHY

## AN OUTLINE OF THE GEOMETRICAL PROPERTIES OF CRYSTALS

BY

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## P R E F A C E

DURING the past twenty years Dr. Victor Goldschmidt of Heidelberg has done much to make crystallography an attractive subject. His development of the two-circle goniometer, the gnomonic projection and a new system of symbols which can be read directly from the projection has largely contributed to this end. For many years the writer has used in his classes the methods of Goldschmidt with gratifying results. The chief motive in writing this book arose from the desire to have in the English language a connected elementary statement of crystallography from this point of view. It is hoped that the present work may be generally useful, not only as a presentation of the geometrical properties of crystals, but also on account of its following in outline the newer methods so splendidly developed by Goldschmidt. I fear I may be criticised for my frequent repetition of some of the fundamental ideas such as are usually only slowly comprehended by the student. My defense lies in the fact that it is necessary for all but the exceptional student.

The extracts from papers on crystallography contained in the last chapter should serve to indicate to the student the chief problems of crystallography as well as the methods of solving them.

The writer acknowledges his great indebtedness to Dr. Goldschmidt for assistance in many ways; to Professors E. H. Kraus, T. C. Chamberlin and R. Salisbury for permission to use figures contained in their works; to colleagues who kindly consented to the publication of extracts of their papers in the last chapter and to Professor A. L. Parsons and H. V. Ellsworth for assistance in proof-reading and in many other ways.

UNIVERSITY OF TORONTO,  
June, 1914

T. L. WALKER



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# CRYSTALLOGRAPHY

## CHAPTER I

### GENERAL PROPERTIES

When a substance such as alum, sugar or common salt is ground finely and put into water it disappears and instead of water and small particles of solid substance we obtain a liquid mixture of the water and the substance — a solution. The solid has become liquid. If such a solution be set aside in an open vessel so that the water may gradually evaporate we find that the solid substance separates again. This newly formed substance on careful examination will be found to occur in the form of numerous little masses bounded by plane surfaces. The shape of the small masses of salt is quite different from that of alum while both differ in this respect from sugar. Those little masses bounded by plane surfaces are known as crystals.

Usually different substances give rise to crystals of different form so that frequently substances otherwise of similar appearance may be distinguished from one another merely by a close examination of the crystals.

On adding finely ground powder of alum and salt and sugar to a beaker of water so that the whole dissolves we obtain one solution which contains all three substances. Now if the vessel containing this solution be set aside and allowed to evaporate gradually the three substances will separate in the form of crystals which are remarkable for their purity. The process involved is known as crystallization, while the substance so separating is said to crystallize.

While we are able to produce crystals of many different kinds by laboratory methods it is not only in this way that crystals are formed. The hard rocks of the earth's crust

are largely composed of the crystallized substances known as minerals. One of the commonest of these, the mineral quartz, attracted the attention of the Greek philosophers many centuries ago. Quartz is as clear as water and frequently quite transparent, and was regarded by Greeks as a peculiar form of ice, possibly resulting from extremely low temperatures. In their opinion ice formed in this way was able to exist through the heat of the summer without melting. This substance they named krystallos — which means ice. From this faulty observation of the Greeks we obtained the word crystal. It is well known now that quartz is not composed of the elements common to ice and water. It is an oxide of silicon and is represented by the chemical formula  $\text{SiO}_2$ .

## CHAPTER II

### FORMATION OF CRYSTALS

When a substance passes from the liquid or gaseous state to a solid condition there is usually a strong tendency toward the formation of crystals. If the substance be chemically simple one kind of crystals may arise — if complex, several kinds. In the laboratory crystals may be formed with no great difficulty according to three distinct methods:

**1. From Solution.** If a substance be dissolved in water or any other liquid and set aside so that this liquid may evaporate, crystals are gradually formed. The amount of a particular substance which can be held in solution by a given solvent varies with temperature. Usually the higher the temperature the greater the solvent power of the liquid in question. At 0° C. 100 grammes of water will dissolve 13.3 grammes of saltpetre, while at 100° C. the same amount of water dissolves 246 grammes. This is important in studying the formation of crystals. As a result crystals may be readily prepared by setting aside a hot saturated solution which with falling temperature gradually forms crystals of the excess of the substance which at the lower temperature it is no longer able to retain in the liquid form.

Crystallization from solution is a process of great importance in the industries, particularly on account of the high degree of purity which is characteristic of the substance which has been crystallized one or more times. If a slightly impure salt be dissolved and allowed to crystallize the impurity is frequently retained in the solution, while the substance which separates in the form of crystals is of a higher degree of purity. By repeated crystallization the impurity may be still further eliminated.

In nature the solvent is sometimes exceedingly complex as in the magma from which quartz and felspar form in

the igneous rocks. In this instance the temperature of the solution at the time of crystallization is above red heat. Some of the substances commonly employed as solvents are alcohol, benzene, carbon bisulphide, water and ether.

**2. By Sublimation.** As a general rule when a solid substance is heated it is converted into a liquid while liquids in their turn at still higher temperatures give rise to gases. There are, however, certain solid substances which do not melt when heated under ordinary atmospheric conditions, but give rise immediately to gases without first appearing in the liquid state. On the other hand gases as a result of cooling or increased pressure normally give rise to liquids which solidify or freeze at still lower temperatures. To these two generalizations there are some exceptions. A solid substance, such as iodine, when heated under ordinary atmospheric pressure passes immediately into the gaseous condition, and if this gas be cooled it returns directly to the solid state usually in the form of crystals. Sulphur is another substance of commercial importance which illustrates the passage from the gaseous to the solid condition. If impure sulphur be heated in a closed chamber it melts and eventually becomes a gas. This gaseous sulphur if led into a cool chamber becomes immediately solid and thereby forms myriads of tiny crystals of exceedingly pure sulphur. A third example of this class of crystallization is represented by the formation of snow. Warm dry air readily takes up water which is transported to various altitudes in the atmosphere. Air highly saturated with water on becoming suddenly cooled below the freezing point gives up part of the water in the solid form as crystals of water or snow. Snow crystals are exceedingly beautiful and always characteristic for the substance from which they are formed. They present the form of a six-rayed star with beautiful plumose shafts (Figure 1).

This method of crystallization may be easily illustrated by placing arsenious oxide, ammonium chloride or iodine in the lower part of an open glass tube held at an angle of 30° and applying a gentle heat. The crystals form on the cooler part of the tube.

**3. From Fusion.** This process of crystallization may be illustrated by the following experiment. Into an earthen

crucible place half a pound of sulphur, heat until the whole mass is fused and set aside to cool until a firm crust has formed on the surface. Break two holes in the crust and pour out the still liquid portion which occupies the interior of the crucible. When the crucible has become quite cold, break with a hammer and observe the growth of crystals on the walls of the vessel. This experiment may be performed with equal advantage by using metallic bismuth instead of sulphur. In both cases the inner lining of the crucible will be found to be splendidly crystallized.

Closely related to this type of crystallization is that which may be observed in the freezing of water or of any other liquid composed of only one chemical substance. Apparently the only difference between the freezing of water and the crystallization mentioned is that sulphur and bismuth are solid at ordinary temperatures while water is liquid. It is necessary, however, to distinguish sharply between the crystallization of sulphur and bismuth from fusion and the crystallization of rock minerals from molten magmas. In both cases the crystals form from hot liquids but it is evident that in the separation of quartz, felspar and mica from a molten rock magma, we are dealing with phenomena to be classed with the growth of crystals of alum or rock salt from water solution. This third process of crystallization from fusion would appear to involve the formation of crystals from a simple chemical individual.

## CHAPTER III

### CHEMICAL PROPERTIES

As a general rule the shape of the crystal is characteristic and more or less distinctive for each chemical substance. By an observation of the crystal form it is easy to distinguish many substances which otherwise have the same general appearance. Alum, sugar, magnesium sulphate and common salt can be readily distinguished in this fashion.

Substances which occur in the form of crystals are usually purer than the same substances in the uncrystallized condition. At one time it was thought that no two substances give rise to crystals of the same form except in the case of those whose crystals belong to the cubic system. Mitscherlich later discovered that arsenates of the alkalies were of the same crystal form as the corresponding phosphates. According to his observations the phosphates and arsenates of sodium, potassium and ammonium occur in crystals of the same character. Since the time of Mitscherlich a great number of such closely related chemical substances have been investigated and it is now well known that regardless of the system of crystallization substances chemically analogous frequently produce crystals which are geometrically identical. A striking example is presented in the case of the alums. Potash alum, the commonest of the series, has the chemical formula  $K_2SO_4 \cdot Al_2(SO_4)_3 \cdot 24 H_2O$ . There is a large number of different alums such as might be obtained by the substitution of sodium, ammonium, caesium, lithium or rubidium for potassium contained in the above formula. Moreover the aluminium may be replaced by chromium, iron or manganese. There are therefore twenty-eight different alums which might be obtained by the simultaneous substitution for both the bases contained in the above formula. These alums may be all represented by the formula  $R^I_2SO_4 \cdot R^{III}_2(SO_4)_3 \cdot 24 H_2O$  in

which  $R^I$  stands for potassium, sodium, ammonium, rubidium, caesium, lithium and  $R^{III}$  for aluminium, iron, manganese or chromium. Such an expression is known as a general formula. Not only are the crystals of all these alums identical in form but all these chemical species may participate in the formation of an individual crystal.

If we suspend a crystal of potash alum in a saturated solution of any alum it will continue to grow without change of form. As some of the alums are colored the zones of different substances may be readily observed by the variation in tint. If several alums be mixed together and a saturated solution prepared from the mixture we obtain on cooling or evaporation very complex crystals in which all the elements contained in the solution are found. The alums present to us, therefore, a series of substances of analogous chemical composition which crystallize in the same form. A crystal of any alum suspended in a saturated solution of any other alum will continue to grow without change of form. Such a series of substances is said to be isomorphous.

In the pyrite group of minerals, isomorphism is splendidly illustrated. The groups of replacing elements are as follows: iron, cobalt, nickel and platinum on the one hand and sulphur, arsenic, antimony, bismuth, selenium and tellurium on the other. The general formula for the whole group is  $RS_2$ , in which  $R = Fe, Co, Ni$  or  $Pt$  and  $S = S, As, Sb, Bi, Te$  or  $Se$ . The chief minerals of the pyrite group are as follows:

Pyrite —  $FeS_2$   
 Smalltite —  $CoAs_2$   
 Chloanthite —  $NiAs_2$   
 Cobaltite —  $CoAsS$   
 Ullmannite —  $NiSbS$   
 Sperrylite —  $PtAs_2$

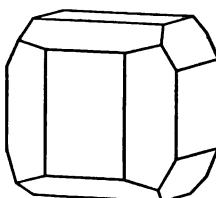


Fig. 2

The minerals of this group commonly crystallize in forms represented by Figure 2.

Groups of chemical elements able to replace one another in isomorphous compounds may be spoken of as isomorphous elements. The student of chemistry will have little difficulty

in selecting examples. Some of the most prominent series of such elements are the following:

1. The alkalis — potassium, sodium, lithium, rubidium and caesium;
2. The alkaline earths — calcium, strontium, barium, to which is frequently added lead;
3. Sulphur, arsenic, antimony, bismuth, selenium and tellurium;
4. Phosphorus, arsenic, vanadium;
5. Iron (FeO), manganese (MnO), calcium, magnesium and zinc.

In view of the isomorphous replacement of closely related elements it becomes necessary for us to modify to a certain extent our conclusion as to the purity of crystals. This replacement frequently occurs not only in artificial substances but also in the crystals occurring naturally in the earth's crust. A mineral containing an element of a certain isomorphous group will frequently be found on careful examination to contain at least small amounts of some of the other elements belonging to that group. This may be illustrated by the following analysis of cobaltite (CoAsS) from Nordmark in Norway.

Cobalt —	$29.17\% \div 58.7 = .497$	atomic ratio	} .609
Nickel —	$1.68\% \div 58.6 = .028$	" "	
Iron —	$4.72\% \div 55.9 = .084$	" "	
Arsenic —	$44.77\% \div 74.9 = .598$	" "	
Sulphur —	$20.23\% \div 32 = .632$	" "	

} 1.230

For comparison with the above the percentage composition of the pure compound may be indicated:

Cobalt —	$35.41 \div 58.7 = .603$	atomic ratio
Arsenic —	$45.26 \div 74.9 = .604$	" "
Sulphur —	$19.33 \div 32 = .604$	" "

On examination, these calculations indicate that in the case of cobaltite, the iron and nickel replace part of the cobalt but always in proportion to the atomic weights of these elements. Similarly it is apparent that some of the arsenic has been replaced by sulphur but here again these elements replace each other in proportion to their atomic weights — 74.9 parts by weight of arsenic are replaced by 32 parts by weight of sulphur.

Isomorphous replacement is the chief cause of variation in composition in the case of crystallized substances. Quartz ( $\text{SiO}_2$ ) is exceedingly constant in its chemical composition apparently because silicon does not take a prominent part in any of the groups of isomorphous elements.

Occasionally a chemical substance appears in two or more forms of crystals which are apparently quite unrelated to one another. Carbon crystallizes in the cubic system as diamond and also as graphite in six-sided plates of the hexagonal system. When a chemical substance appears in two such forms it is said to be dimorphous; when in three such forms, trimorphous. Titanium dioxide which occurs in nature as three distinct minerals — rutile, anatase and brookite — is the best known instance of trimorphism.

## CHAPTER IV

### MATHEMATICAL CHARACTERISTICS

**1. Gnomonic Projection.** A line which is at right angles to a plane surface is spoken of as the normal to that surface. If the direction of the normal be known it follows that the direction of the plane is also known. In the study of crystals, some of which are very complex polyhedra, it is sometimes advantageous to represent the direction of each plane surface

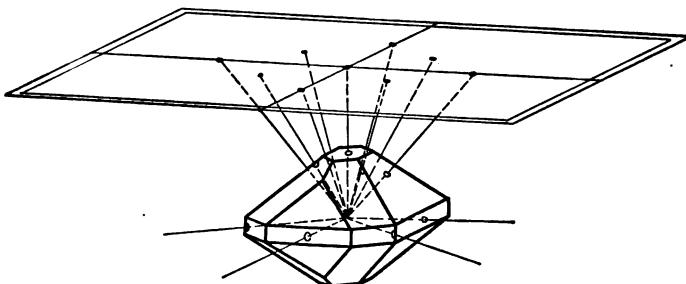


Fig. 3

by its normal. Figure 3 indicates this method of representing the direction of the faces which occur on a crystal of bournonite. Each of the lines stands at right angles to some face and all these lines are so placed as to pass through one central point.

Models representing the direction of normals for any particular crystal can be readily prepared by the use of sharp needles which may be forced into cork which serves to hold them together with the desired direction. Such a bundle of needles is a normal model. The intersection of the normals with a plane would be indicated by a number of points on that plane. The absolute size of the area over which these points would be distributed depends upon the distance be-

tween the plane and the centre of the crystal. The scale is proportional to this distance. Such a plane of points is called a gnomonic projection. In the projection the direc-

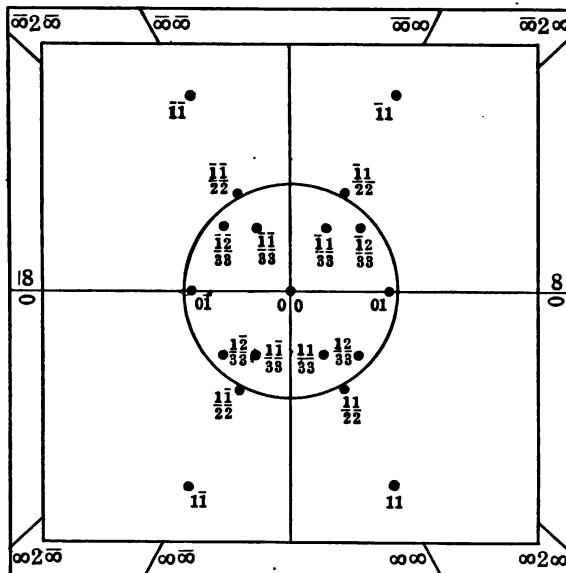


Fig. 4

tions of all the faces of a crystal may be accurately indicated. The amount of information which is conveyed by a gnomonic projection is more or less dependent upon its direction. Ordinarily the plane of projection is horizontal and since crystals are always set up or oriented in a particular way for study it follows that such a projection is at right angles to the accepted vertical direction of the crystal. Figure 4 represents a projection parallel to the base of the topaz crystal shown in Figure 5. The distance between the centre of the crystal and the plane of projection is indicated by the radius of the circle in the projection. For convenience this height is ordinarily five centimeters.

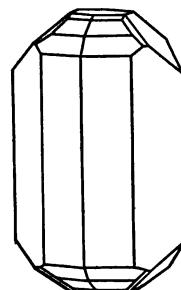


Fig. 5

**2. Regular Arrangement of Projection Points.** A brief examination of this projection suffices to show that the points are distributed in a regular manner. They show a marked tendency to occur in lines. Where three points occur on a straight line it will be found that these points correspond to three faces on the crystal which meet in parallel edges or which would form parallel edges if extended. A series of faces on a crystal intersecting in parallel edges is called a zone and a series of points lying in a straight line in a gnomonic projection may be referred to as a zone line. In this projection of topaz many such zone lines occur. Some of these lines are parallel to other lines. In the topaz projection there are two sets of parallel lines at right angles to one another. If a projection be turned so that one set of parallel lines is right and left then the other will appear fore and aft. Let us examine the position of those parallel lines having the right and left direction. One of them passes through the centre of the picture; others before or behind this central line. If the distance between any pair of these right and left lines be measured it will be found that it is rationally related to the distance between any other pair. Similar relations obtain in connection with the fore and aft zone lines. While the distances between pairs of right and left lines are simply related to one another there is no simple relationship between these distances and the corresponding distances between the fore and aft lines. For topaz these two sets of measurements are always irrationally related to one another. Set a pair of compasses so as to mark exactly the distance between any pair of right and left lines and then measure with this unit the distance of each projection point from the central right and left line. The number of paces will be found to be either a small whole number, a simple fraction, infinity or zero. Similar relationships hold with regard to these points when measured from the central fore and aft line. As has been already remarked in the case of topaz the length of the fore and aft pace is irrationally related to the right and left pace. Some crystals are very complex on account of the number of faces which occur upon them but in all cases only such faces occur as are in accord with the rationality of the

pace numbers. Faces having directions which would give us projection points out of harmony with this generalization do not occur. This first generalization may be referred to as the law of rationality of the pace numbers. Sometimes it is called the law of the rationality of the axial parameters. The propriety of this name will be apparent later. This is the first and most remarkable law of crystallography.

**3. Formation of Crystal Symbols.** In dealing with such complex polyhedra it is not only desirable but necessary to devise some system by which the multitude of faces can be accurately and simply indicated. Many systems of symbols have been suggested. One of these can be derived immediately from the gnomonic projection. Having already arbitrarily selected fore and aft and right and left pace lengths let us determine the number of such paces forward or backward involved in any projection point. If the point lie in front of the right and left medial line the number may be indicated as positive; if behind, negative. Next let us determine the pace numbers in the same fashion in regard to the fore and aft medial line. Measurements to the right are positive; measurements to the left, negative. Let it be agreed that the symbol for any face consists of the fore and aft pace number with its appropriate sign, followed by the right and left pace number similarly indicated. These projection symbols were first suggested by Prof. Victor Goldschmidt and are commonly known as the Goldschmidt symbols. Goldschmidt has introduced a further abbreviation in that when the two pace numbers are identical the second is suppressed, thus  $00 = 0$ ,  $\infty\infty = \infty$ ,  $11 = 1$ ,  $33 = 3$ . The system of symbols popularized by Professor Miller and commonly known by his name can be readily obtained from the projection symbols by placing the figure 1 after the projection symbol and then simplifying by dividing or multiplying so as to get rid of fractions and  $\infty$ . As Miller's system has been accepted in all parts of the world it is desirable to be able readily to transform any other system which may be employed into this international system. In the descriptive part of this book it is proposed to regularly use both the systems of symbols outlined. The derivation of symbols from the gnomonic projection is not always so simple as it is in the

case of topaz, but on the whole comparatively little effort is required to obtain a fair facility in this short hand descriptive method.

On the projection of topaz the points are so distributed as to be symmetrical about the two medial lines which stand at right angles to one another. If a normal be dropped from point  $11$  to the fore and aft line and produced an equal distance beyond the point of intersection we observe that it reaches the point  $\bar{1}\bar{1}$ . These points are said to be symmetrically arranged with regard to the fore and aft line. The points  $11$  and  $\bar{1}\bar{1}$  are also symmetrically placed with regard to the right and left line. It may be noted that the occurrence of a point which is not contained in either of the lines of symmetry necessarily implies the occurrence of three other points — the symmetry about two lines at right angles to one another requires this. On the other hand, a projection point which lies on one of these lines of symmetry will necessarily be accompanied by a second point, the two being placed symmetrically about the other line. The third possible condition consists in the occurrence of the projection point at the intersection of the two lines of symmetry. Such a point may occur alone. It appears, therefore, that on crystals there are certain sets of faces all of which may be expected when any one of them occurs. In the projection in question the occurrence of the face  $11$  requires the simultaneous occurrence of  $1\bar{1}$ ,  $\bar{1}1$ , and  $\bar{1}\bar{1}$ . Such sets of faces are known as crystal *forms*. As indicated in the topaz projection there are certain forms which are there represented by four faces, others by two and one by one.

There is another element of symmetry well represented in the topaz projection. If a line be drawn from any projection point to the point where the two lines of symmetry meet, and produced an equal distance beyond it will always terminate in a similarly situated point. This peculiarity of the projection is said to be due to its symmetry about its central point. The degree of symmetry as exhibited in projections varies with that of the crystal. The topaz projection is symmetrical about two lines at right angles to each other and also about its central point. This arrangement of projection points with reference to certain lines and points of symmetry is so regular that it has been formulated as

the law of symmetry. *Crystal faces occur in sets of such numbers and with such inclinations as to preserve the symmetrical arrangement with regard to the point, lines and planes of symmetry which are characteristic for the crystal individual in question.*

There are many substances which crystallize in accordance with the same symmetry exhibited by topaz, but if we were to classify in accordance with symmetry all the crystal individuals which have been carefully measured and examined, it would be necessary for us to provide about thirty distinct classes. Some substances crystallize so as to exhibit a large number of planes of symmetry. Generally the higher the number of planes of symmetry the larger the number of faces comprising a crystal form. The diamond possesses a very high degree of symmetry and occasionally its crystals present to us forms involving forty-eight faces. As will be seen later the classification of crystals is based upon symmetry.

The distance measured on this gnomonic projection, from the central point to the various projection points belonging to any one set, is constant. This is due to the fact that the angles of the crystal formed by these several pairs of faces are identical. This would be true without regard to the size of the crystal, the locality from which it is obtained or even the relative sizes of the faces. This regularity which is so plainly shown in the gnomonic projection as to be scarcely worthy of mention was observed in 1669 by Nicholas Steno who first formulated for us the law of the constancy of interfacial angles. *Under given physical conditions and for a particular crystal individual the angle between corresponding pairs of faces is constant.*

Before assigning symbols to the projection points we found it necessary to arbitrarily select fore and aft and right and left paces. The ratios existing between the fore and aft pace, the right and left pace and the height of the plane of projection above the centre are constant and characteristic for the mineral topaz. For this type of crystal these three values are irrationally related to each other. There are many other substances presenting the same symmetry as topaz for which similar projections might be formed, but it is very improbable that for any other substance would the

ratio between these three values even closely approximate the ratios shown by topaz. As it will be necessary to refer frequently to these ratios let us indicate for the sake of brevity the fore and aft pace as  $p_0$ , the right and left pace as  $q_0$  and the height of the plane of projection as  $r_0$ . The ratio  $p_0: q_0: r_0$  for a substance such as topaz is as characteristic for it as its specific gravity, its index of refraction or its coefficient of expansion. These ratios are called crystallographic constants. Later we must study the methods involved in the exact mathematical determination of these ratios. For the present  $p_0$  may be found by measuring carefully from point to point forward and backward. The same method may be applied to  $q_0$ . If the projection were prepared at a height of five centimetres above the centre of the crystal, then  $r_0$  is equal to five centimetres. For topaz the values obtained from these measurements should be as follows:  $p_0: q_0: r_0 :: 9.0245: 4.7695: 5$ . Now if we simplify making  $r_0$  unity the ratio becomes  $p_0: q_0: r_0 :: 1.8049: .9539: 1$ . This is a graphic method for determining from the gnomonic projection the approximate values of the crystallographic constants.

The individuality of crystallographic constants may be readily observed by an examination of the following table in which the characteristic values are indicated for several substances having the same symmetry as topaz:

	$p_0$	$q_0$	$r_0$
Anglesite	1.6421	1.2894	1
Anhydrite	1.1204	1.0008	1
Cerussite	1.1853	.7230	1
Celestite	1.6426	1.2830	1
Topaz	1.8049	.9539	1

The graphic determination must not be regarded as merely suitable for purposes of demonstration. From a carefully prepared gnomonic projection values may be obtained always accurate to the first three decimals. Naturally the most accurate results are to be obtained by mathematical calculation.

**4. Measurement of Crystals.** The preparation of a gnomonic projection implies very accurate measurement of the crystal to be represented. Various instruments known as

goniometers have been devised for crystal measurement. The earlier instruments were of simple construction, easily operated, and correspondingly inaccurate. During the last one hundred and thirty years many improvements have been made so that it is now possible to measure the best crystals with an accuracy which permits variations of only a fraction of a minute. The instruments of the earlier period were scarcely designed for an accuracy greater than one-half degree. These goniometers fall into two classes. The earlier instruments are known as contact goniometers and were particularly useful for the examination of large crystals regardless of the roughness or smoothness of the crystal face. The more modern instruments are known as reflecting goniometers and give good results with crystals half a millimetre in diameter, provided the faces are bright so as to reflect light. In the following paragraphs some of the chief types are briefly sketched.

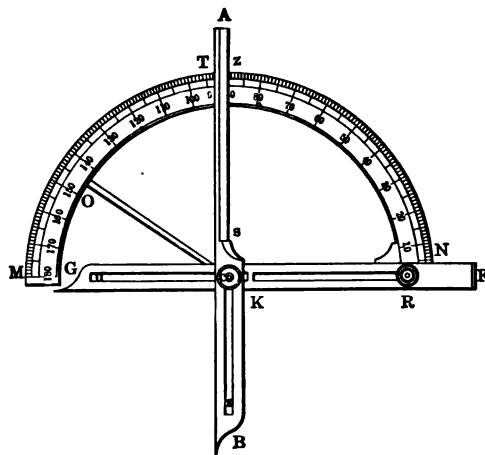


Fig. 6

(a) *The Contact Goniometer of Carangeot.* The first instrument for the measurement of solid angles was devised by Carangeot in Paris in 1780 and was used by him in the making of crystal models from clay. It consists of a graduated semicircle to the centre of whose diameter is attached a swinging bar. A crystal whose angle is to be measured is carefully adjusted between the two bars. The distant end of the movable bar indicates the magnitude of the crystal angle on the graduated semicircle. It is necessary in making such measurements to be careful that the lines of contact for the two bars on the crystal surfaces are at right angles to the edge between the faces. If this be not kept in mind very variable angles may be obtained. When we say that the

angle between a pair of faces is  $120^\circ$  we mean that a pair of lines, drawn from a point on the edge between them and at right angles to the edge, form an angle of  $120^\circ$  with one another. The goniometer of Carangeot is in use at the present day and has undergone very little modification.

(b) *Wollaston Reflecting Goniometer*. The first reflecting goniometer was prepared by Wollaston in 1809 and consisted of a graduated circle movable about a horizontal axis. A crystal whose interfacial angle is to be measured is attached to this

axis in such a way that the edge between the faces forming the angle is parallel to the axis. The crystal faces must be brilliant reflectors. In a dark room a light is placed so that the reflection from one of the faces reaches the eye of the observer. If the disc be revolved the reflection from the other face will not reach the eye until the second face as a result of the revolution assumes a position parallel to that of the original reflecting face. The angle through which the crystal must be revolved will represent the true interfacial angle, providing the plane including the incident

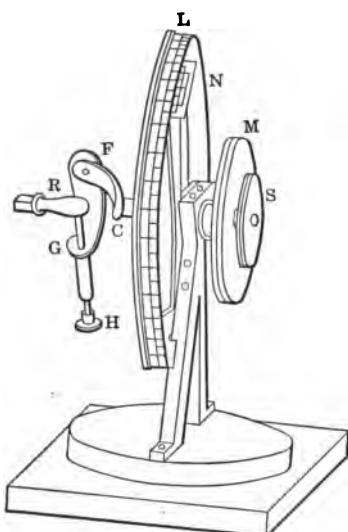


Fig. 7

and reflected rays is at right angles to the edge of the crystal. With the earlier reflecting goniometers it was difficult to be sure that this plane was always *exactly* at right angles to the edge of the crystal.

By the use of the old style contact goniometer the inner angle was measured so that on a six-sided prism the angle would be spoken of as  $120^\circ$ . With the reflecting goniometer it is not necessary to turn the crystal through  $120^\circ$  to secure a reflection from the second face but only through

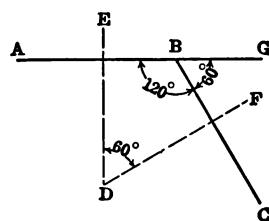


Fig. 8

60° which is the supplement of the angle as observed when measurement has been made with the contact goniometer. These angles are known as inner and outer angles. The propriety of these terms is illustrated by Figure 8.

If the light be just reflected from the face AB and the crystal be turned till BC attains the direction at first possessed by AB, the second reflection will occur. The angle through which the crystal must be turned is represented in the figure EDF, which is equal to the outer angle CBG. In the following pages the interfacial angles refer — unless otherwise specially mentioned — to the outer angles.

(c) *A Modern Reflecting Goniometer.* In this instrument the graduated circle is horizontal and the instrument is supplied

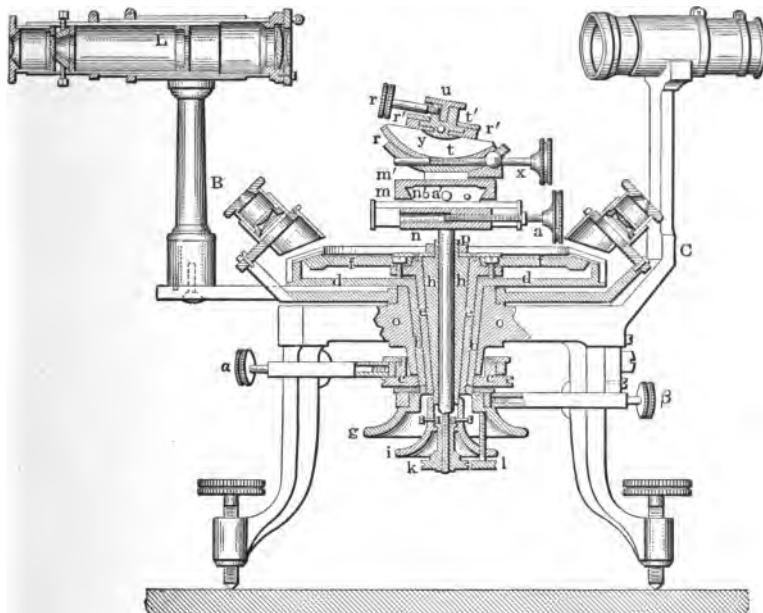


Fig. 9

with a collimator through which the light is led to the crystal, while a telescope carries the reflected ray from the crystal to the eye. The crystal is supported on a bar which rises from the centre of the graduated disc and must be so placed that the reflecting face lies at the point where the axes of the

collimator, telescope and graduated circle meet. Moreover it is necessary to adjust or orient the crystal so that the edge over which the angle is to be measured is normal to the plane containing the axes of the collimator and the telescope. When this adjustment has been made the inner angle may be determined by bringing these two faces in succession to the point of reflection, reading the graduated circle for each position and noting the difference. This instrument, like all reflecting goniometers, must be operated in a dark room with a special source of light placed in line with a collimator. This instrument is a modern example of a one-circle reflecting goniometer. Many mechanical attachments are introduced to facilitate the exact orientation of the crystal and to secure such a reflection as to render accurate reading possible (Figure 9).

(d) *Two-circle Contact Goniometer.* This instrument consists of a horizontal graduated circle from whose centre rises a rod to which the crystal may be cemented. A vertical graduated circle having the crystal approximately at its centre is also provided. The crystal to be measured is attached to the rod with the ordinary standard orientation and then by means of rotation of the horizontal disc and by adjustment of the sliding rod which serves as a radius of the vertical circle each crystal face in turn is brought into such a position that its normal coincides in direction with the movable rod, that is, the small circular disc on the end of this radial rod is parallel to the crystal face. The vertical circle is so graduated that the zero is at its highest point and from that zero point the

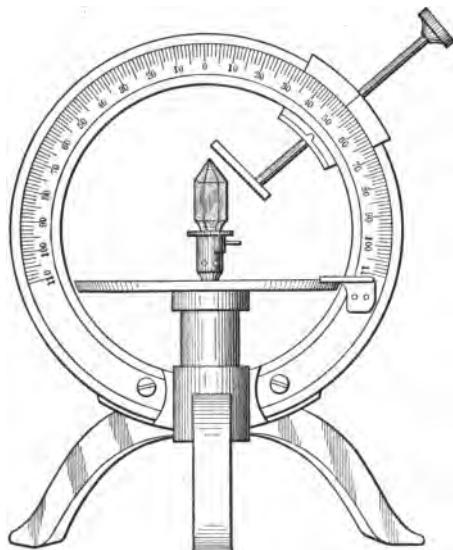


Fig. 10

numbers increase both ways to a maximum of  $90^\circ$ . This is a simple style of two-circle contact goniometer. There are two angles to be read for each crystal face. By this method is obtained a record of the direction of each face normal. One set of measurements corresponds in a way to longitude. These are derived from the reading of the horizontal circle which on this instrument reads from zero to  $360^\circ$  while longitude ordinarily numbers in both directions and attains a maximum of  $180^\circ$ . A second set of measurements corresponds to latitude but again in a reverse order. The latitude of the pole on an earth globe is  $90^\circ$ , the latitude of the equator being zero. In the reading derived from this two-circle goniometer the latitude of the pole is zero and of the equatorial direction  $90^\circ$ . This instrument (Figure 10) and the one about to be described were devised by Prof. Victor Goldschmidt and are among the most suitable goniometers for didactic purposes.\*

(e) *Two-circle Reflecting Goniometer.* The instrument represented in Figure 11 consists of two graduated circles, one of which is horizontal and parallel to the plane of the collimator and the telescope. The second is borne by an arm in such a way that it is vertical and revolves about a horizontal axis, lying in the plane of the collimator and telescope, while the axes of the collimator, telescope, horizontal and vertical circles meet in one central point. The arm which carries the vertical circle swings freely about the axis of the horizontal circle. An arm similar to that which carries the vertical circle supports the telescope, which is clamped when the instrument is being used for crystal measurement. The crystal to be measured is attached to the axis of the vertical circle in such a manner that the standard vertical direction of the crystal coincides with the direction of this axis. For measurement it is necessary that the crystals should be centred so that the reflecting face is very near the point of junction for the axes of the telescope, the collimator and the vertical circle. This instrument is in many respects similar to the last. In measurement we get two sets of

\* The instruments represented in figures 10 and 11 are manufactured by Fritz Rheinheimer, Landfriedstr. 6, Heidelberg, Germany.

angles, the readings from the horizontal circle coincide with those from the vertical circle of the contact goniometer and the readings from the vertical circle correspond with those from the horizontal circle of the contact goniometer. Very accurate measurements can be made with this instrument. It has a great advantage over the ordinary one-circle reflecting goniometer in that it is possible to read with one setting up of the crystal all the faces which are ordinarily necessary. With a one-circle reflecting goniometer it is necessary to orient afresh the crystal for each zone of faces to be measured. In the case of the very small crystals this frequent orientation may become very tedious.

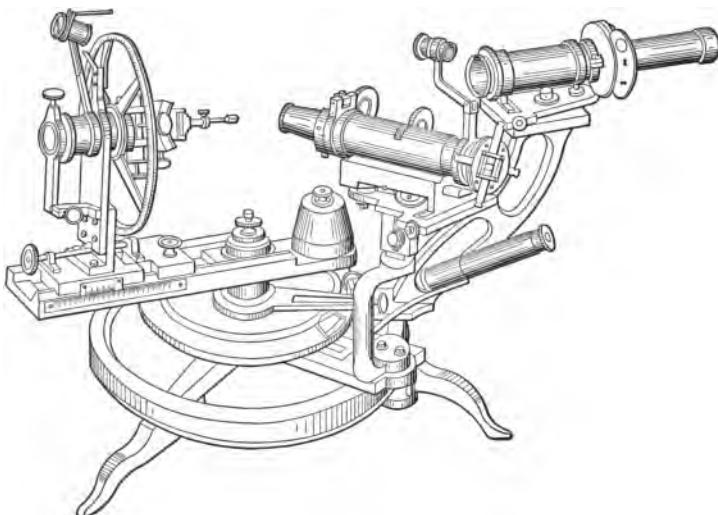


Fig. 11

**5. Preparation of Gnomonic Projection from Measurements.** The measurements obtained by the use of the two-circle goniometer imply first a polar direction corresponding in a way with the pole of the earth as represented in the earth globe and second, a plane of first meridian or a plane of reference to which are referred those angles corresponding to longitude. In the case of crystals the pole coincides with the standard vertical direction of the crystal while the plane of the first meridian as shown by the measurements is merely

a chance direction. It is in fact that plane through the crystal which contains the axis of the vertical circle and the zero point upon that circle. Starting with these measurements the present problem is the preparation of a gnomonic projection. Let the plane of projection be at right angles to the pole, which coincides with the standard vertical direction of the crystal, and five centimetres from the central point of the crystal where all the normals may be supposed to intersect. It is equally easy to prepare such a projection at any other distance from the centre, but if the distance be less than five centimetres the points are so close together that any error in plotting is relatively more serious, while if the distance from the centre be greater the projection becomes unnecessarily large without securing any compensating advantage. In attempting to make such a projection we should be provided with drafting paper on which two lines have been drawn at right angles to one another. On paper specially prepared for such projections a network of lines parallel to these two are arranged at intervals of one centimetre with further subdivision in lighter lines one millimetre apart. Enclosing this whole netted area is a graduated circle whose centre lies at the intersection point of two of the heavier lines. This circle is graduated from zero to  $360^\circ$ , the numbers increasing in the direction of the hands of a clock. The zero point for this graduation is at the right end of the right and left line passing through the centre. Various sheets of paper intended for this purpose are prepared and may be purchased with all the preliminary lines upon them so that it is not necessary to spend time upon preparing the paper prior to making the projection. With such a sheet of paper the preparation of a gnomonic projection from measurements is very simple. Take a particular face whose readings are  $31^\circ 30'$  on the horizontal circle (corresponding to latitude) and  $217^\circ 20'$  on the vertical circle (corresponding to longitude). The pole of the projection is marked by the point at the centre of the circle. The projection point for this face lies

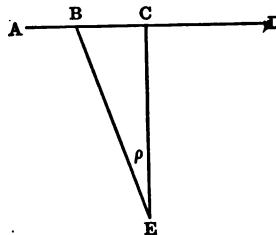


Fig. 12

on a line connecting the centre with the reading  $217^{\circ} 20'$  on the projection sheet. To find its position on that line it is only necessary to measure a length from the centre equal to five times the tangent of the angle of the horizontal circle in centimetres. This angle corresponding to latitude is ordinarily designated by the letter  $\rho$ . The diagram (Figure 12) represents a section through the plane of projection, the centre of the crystal and the projection point for the face in question. It is readily apparent from an examination of this diagram that the distance of a projection point from the pole is represented by the product of the tangent of  $\rho$  and the height of the plane of projection above the centre of the crystal.

E = crystal centre

AD = plane of projection

B = projection point

$\angle BEC = \rho$

CE = h = height of plane above centre

BC = distance of projection point from pole

$$\frac{BC}{CE} = \tan \rho, \text{ also } \frac{BC}{h} = \tan \rho$$

$$\therefore BC = \tan \rho \cdot h.$$

By this method the various projection points may be located and the projection completed. Where the value of  $\rho$  exceeds  $80^{\circ}$  the distance of the projection point from the centre may be so great as to fall outside the bounds of the paper. Such cases are unusual except where  $\rho$  is  $90^{\circ}$ . In this case the intersection of the plane by a normal with  $\rho = 90^{\circ}$  will only occur at  $\infty$  so that it is usual to indicate projection points of this class of face by means of an arrow on the border of the sheet of paper, the arrow lying on the line joining the centre with the projection point.

If a projection prepared in this way be examined as to the direction of its lines of symmetry it will usually be found that none of these lines have the right and left and fore and aft direction but that it is necessary to revolve the projection through a certain angle in order to secure this arrangement. As has already been stated there is a standard vertical direction arbitrarily agreed upon for each type of crystal. The

standard orientation for each crystal also prescribes right and left and fore and aft directions. In the case of the rhombic system, to which topaz belongs, one of the lines of symmetry must be right and left and the other fore and aft. A simple calculation, which need not be outlined here, is necessary to determine the number to be subtracted from the readings obtained on the vertical circle so as to give the crystal its standard orientation in the projection. This angle through which the projection must be revolved or which must be subtracted from or added to the readings of the vertical circle is known as  $v_0$  according to Goldschmidt.

Figure 4 represents a gnomonic projection for topaz on which the forms present on Figure 5 are indicated.

**6. Mathematical Determination of Crystal Symbols and Constants.** The method to be pursued in the graphic determination of symbols and constants has been already indicated. As will be readily apparent the graphic method does not give the most accurate results. There are two sources of error. First, the projection point may be placed a short distance from its theoretical position. Second, when we attempt to measure the distance between points it is difficult to read distances less than one-half millimetre. More accurate results may be obtained from the crystal measurements by mathematical calculation. The projection of topaz falls into four quadrants. In each quadrant similar sets of points are present. This is in accordance with symmetry about two lines at right angles to one another. Before proceeding to mathematical calculation it is usual to still further recast the readings obtained in the vertical circle so as to indicate the angle between the plane of reference and the plane containing the pole and the normal, for the face. To illustrate, suppose there are four faces belonging to the same crystal set or form having the following vertical readings:  $37^\circ 28'$ ,  $142^\circ 32'$ ,  $217^\circ 28'$ ,  $322^\circ 32'$ . One projection point falls into each quadrant. The plane carrying the normal for the second face makes with our plane of reference an angle of  $37^\circ 28'$  but in the second quadrant. The plane carrying the normal for the third face lies in the third quadrant and makes an angle with the plane of reference of  $37^\circ 28'$  and similarly for the fourth face. This angle  $37^\circ 28'$  is charac-

teristic therefore not only of the first but also of the second, third and fourth faces. It is designated by the Greek letter  $\phi$ . Before proceeding to the calculation of the symbols and constants it is necessary to first determine  $v_0$  which may be subtracted from the readings for each face on the vertical circle and afterwards to determine the angle  $\phi$  which locates each point in its particular quadrant. Our problem therefore is to calculate the distance of a point from the right and left line of reference and from the fore and aft line of reference.

These values are functions of  $\phi$ ,  $\rho$  and the height of the plane of projection above the centre. If the last value be made unity then the fore and aft distance ( $x$ ) from the right and left line is  $\sin \phi \cdot \tan \rho$  while the right and left distance ( $y$ ) is the  $\cos \phi \cdot \tan \rho$  (Figure 13). The derivation of these formulae is indicated as follows:

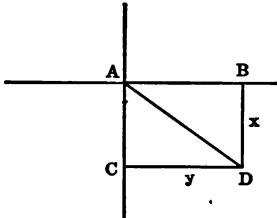


Fig. 13

To calculate the lengths of  $x$  and  $y$  for the face 11 as shown in the gnomonic projection (Figure 4) from D draw DB and DC normals to the lines of symmetry and DA to the common point of these two lines. Let the height of the plane of projection above the crystal centre be unity

Then  $AD = \tan \rho$

$\angle BAD = \phi$

$$\frac{BD}{AD} = \sin \phi \quad \therefore \frac{x}{\tan \rho} = \sin \phi$$

and  $x = \sin \phi \cdot \tan \rho$

$$\text{Similarly } \frac{AB}{AD} = \frac{CD}{AD} = \frac{y}{\tan \rho} = \cos \phi$$

$$\therefore y = \tan \rho \cos \phi$$

By this method the lengths are calculated mathematically. The values of  $x$  for the various faces are found to be simply related to one another. They are easy multiples of some common number. That common number for all the values of  $x$  is  $p_0$ . For any particular face the number of paces ( $p$ ) may be obtained by dividing  $x$  by  $p_0$ . Similar relations hold between  $y$ ,  $q_0$  and  $q$  — the right and left pace number.

These calculations for the various faces may be made without any difficulty for each face but when dealing with complex crystals presenting a large number of faces it is an advantage to record the results in some definite form such as the following:

TABLE ILLUSTRATING AN ORDERLY METHOD OF RECORDING CALCULATIONS

Face No.	Vertical	$\phi$	$\rho$	(x) = $\sin \phi \tan \rho$	$p_0$	p	Miller's symbol.	Goldschmidt's symbol.
				y = $\cos \phi \tan \rho$	$q_0$			
1	62° 08'	62° 08'	63° 54'	1.8049 .9539	1.8049 .9539	1 1	111	1
2	242° 08'	62° 08'	76° 14'	3.6098 1.9078	1.8049 .9539	2 2	221	2
3	97° 32'	82° 28'	61° 13'	1.8049 .2385	1.8049 .9540	1 1	414	11
4	180°	0°	43° 39'	zero .9539	.9539	0 1	011	01
5	270°	90°	42° 04'	.9024 zero	1.8048 0	1 0	102	10
6	32° 14'	32° 14'	73° 32'	1.8049 2.8617	1.8049 .9539	1 3	131	13

There are two problems either of which may be solved from such calculations but both cannot be solved simultaneously. We find the length of the distance x or y but as

$$x = p \cdot p_0$$

$$y = q \cdot q_0$$

it is not possible to determine p unless  $p_0$  be already known or to determine q unless  $q_0$  be known. If on the other hand p and q be known  $p_0$  and  $q_0$  may be determined. If we measure a crystal for which the constants  $p_0$  and  $q_0$  have been determined then we may wish to determine the values p and q and in this way find out the symbol for any face.

On the other hand we may measure such a crystal in order by careful measurements and averages to check still further the values  $p_0$  and  $q_0$ . In working with crystals for which the symbols and constants have not already been fixed there must be an arbitrary choice of certain faces to which symbols are assigned and then all the other faces must receive symbols in accord with these. The values assigned to  $p_0$  and  $q_0$  are based on the averages for the values calculated from the best faces.

**7. Stereographic Projection.** The gnomonic projection is not commonly employed for purposes of illustration in works of reference on mineralogy and crystallography. Another type of projection more generally employed known as the stereographic projection is obtained in the following manner. Suppose a crystal to be placed at the centre of a sphere. From the central point the normals for the various faces radiate and intersect the surface of the sphere. The eye of the observer is placed at a point on the sphere in line with the standard vertical direction of the crystal. A plane passing through the centre of the crystal and at right angles to the standard vertical direction is the plane of the stereographic projection. On this plane the *apparent* position of the various

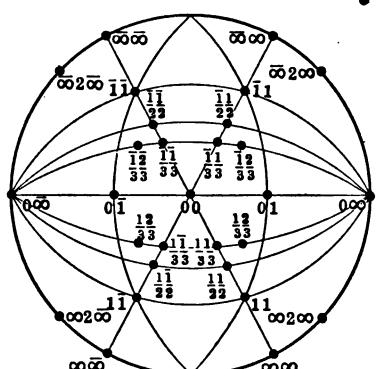


Fig. 14

points on the sphere surface is indicated. All the projection points on the upper half of the sphere are contained within a circle, corresponding to the equator. Those on the half of the sphere toward the observer lie outside that circle. The complete stereographic projection takes account of the points within and without but in general use the points lying outside the circle are not indicated. Faces belonging to the

zone which intersects in edges parallel to the standard vertical direction are represented by points on the circle referred to. For the rhombic system the face 001 will appear in the centre of the circle. In this type of projection the zone lines appear in

three forms: (1) complete circle; (2) arcs of circles; and (3) straight lines, where the zone contains the pole. The magnitude of the projection is proportional to the radius of the sphere. Stereographic projections may be prepared by the same method followed for the gnomonic projection. The chief difference consists in the fact that the distance of the projection point from the centre of this projection is represented by  $\tan \frac{\rho}{2}$  times the radius, while for the gnomonic projection the distance is  $\tan \rho$  times the height of the plane of projection above the crystal centre. Figure 14 is a stereographic projection of topaz exhibiting the projection points for the faces shown on Figure 5. The derivation of the formula used is indicated by the following diagram (Figure 15):

Let E represent the centre of the crystal placed at the centre of the sphere, AC the section through the plane of the gnomonic projection, DHF a section of the sphere DF the plane of the stereographic projection and H the point of view for the stereographic projection. EGK is a face normal making the angle  $\rho$  with the pole HEB. The normal intersects the sphere surface at G and the plane of the gnomonic projection at K. Its intersection point at G will appear to lie on the stereographic projection at the point L.

In the  $\triangle$  EGH, EG = EH

$$\therefore \angle EGH = \angle EHG$$

$$\text{But } \angle \text{ BEG} = \angle \text{ EGH} + \angle \text{ EHG}$$

$$\therefore \angle EHG = \frac{\rho}{2}$$

The distance of the point L from the centre of the projection is represented by the distance EL.

$$\frac{EL}{EH} = \tan \frac{\rho}{2} \therefore EL = \tan \frac{\rho}{2} \cdot \text{radius}$$

**8. Co-ordinate Representation of Face Direction.** It is only in recent years that the gnomonic projection has become

fundamental in the study of crystallography. Previous to this the inclination of a crystal face was indicated by reference to three lines (four in the case of the hexagonal system) intersecting in one point. Such a system of imaginary lines is known as the axial cross or axes of reference. For the cubic, tetragonal and rhombic systems these three lines are at right angles to one another. In the monoclinic system two of them are at right angles to one another, the third being at right angles to one of the first pair but oblique with regard to the other. In the triclinic system all the angles are oblique. For the cubic, tetragonal and rhombic systems the axes are so oriented that their directions are vertical, right and left, and fore and aft. The direction of a plane is fixed when we know the relative lengths measured from the central point at which it intersects the three axes. In the rhombic, monoclinic and triclinic systems the planes intersect the three axes in such a manner that the lengths on the three axes measured from the centre to the points of intersection are always irrationally related to one another. The three axes are said to be unequal. This corresponds to the irrational relationship which we have already observed for the rhombic system in the case of the values  $p_0$ ,  $q_0$  and  $r_0$ . In certain systems it is possible for a plane to intersect two axes so that the lengths of the intercepts are rationally related. Such axes are equivalent. In the cubic system all three axes are

of this character. For the rhombic, monoclinic and triclinic systems, the fore and aft axis is designated by the letter  $a$ , the right and left by  $b$ , and the vertical axis by  $c$ . Opposite ends of these axes are indicated by plus and minus signs as may be observed in the diagram (Figure 16).

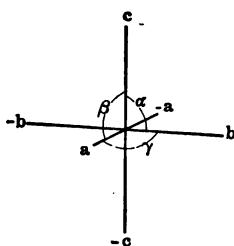


Fig. 16

To the angles between the various pairs of axes are assigned the letters  $\alpha$ ,  $\beta$  and  $\gamma$ .

In the rhombic these are all  $90^\circ$ . In the triclinic system they are all oblique, while in the monoclinic only  $\beta$  is oblique.

If several faces of a crystal intersecting the same ends of three axes be moved parallel with themselves so as to pass through the same point on one of the axes it will then be

found that the lengths cut off on either of the other axes by the various faces are simply and rationally related to one another. Only those faces are possible on a crystal which have such peculiar directions as to be in accord with this generalization, known as the law of rationality of axial parameters. This limitation as to the possible intersections of the axes of reference also finds expression in a very simple form in the rationality of the pace numbers as observed in the gnomonic projection.

**9. Derivation of Miller's Index Symbols.** In all crystals except those belonging to the cubic system it is necessary to select arbitrarily some unit crystal form or *ground form* before it is possible to proceed to assign symbols to the other faces that are present or to calculate crystallographic constants. In determining symbols in a gnomonic projection this arbitrary choice was made by the selection of a particular projection point whose distances from the right and left and fore and aft lines were regarded as the unit paces  $p_0$  and  $q_0$ . Similarly when a complex crystal is studied with regard to the axes of reference a certain form whose faces intersect the three axes is arbitrarily selected as the unit or ground form. In the rhombic system that form has eight faces and is known as the rhombic pyramid. Any other form whose faces intersect three axes in this system is a pyramid but the ratios of the axial intersections differ for the different pyramids. The face directions of crystals are constant though the size of the face is exceedingly variable. In indicating the ratio of the lengths of the intercepts for the unit pyramid of the rhombic system it is customary to make the length on the  $b$  axis unity. For topaz the ratio is  $a:b:c: .5285:1:4769$ . The calculation of the constants  $a:b:c$  usually involves the use of spherical trigonometry. These *axial ratios* may be derived from the *polar elements*  $p_0:q_0:r_0$ . For the rhombic

system the following values are sufficient:  $a = \frac{q_0}{p_0}$ ;  $b = 1$ ;  $c = q_0$ . The transformations for the various systems will be indicated later.

In Figure 17 three pyramidal faces are shown intersecting the axes in various ratios. One of them has such a direction that while intersecting the  $a$  axis at the same point as

the unit pyramid it intersects the  $b$  axis at twice and the  $c$  axis at three times the distance characteristic of the unit pyramid or ground form. The direction of this face may be indicated according to the method of Weiss as follows, 1a:2b:3c. For the third pyramid the corresponding values are 2a:1b:2c. Planes which intersect only two axes at finite distances intersect the third at an infinite distance. The selection of the ground form determines the axial ratios for a particular crystallized substance. The characteristic element in the symbols of Weiss is the number placed before the letters a, b and c. These numbers are known as parameters. They are always rationally related to one another. The index symbols of Miller are the simplified reciprocals of these parameters written in the same order as in the symbols of Weiss. Thus for 2a:1b:3c the parameters are 2.1.3 whose reciprocals  $(\frac{1}{2}, \frac{1}{1}, \frac{1}{3})$  when simplified by multiplying by the least common denominator (6) gives Miller's symbol 362. These new values are known as indices and 362 is the symbol for only one face. If the face symbol be enclosed in brackets (362) it becomes a general symbol for a whole form, which in the rhombic system is a pyramid with eight faces. By placing upon the index number the appropriate sign, plus or minus, to designate the end of the axis cut it is possible by this system to indicate each

of the eight faces belonging to the crystal form in question. The eight face symbols of this form are 362,  $\bar{3}62$ ,  $\bar{3}62$ ,  $\bar{3}62$ ,  $3\bar{6}2$ ,  $3\bar{6}2$ ,  $\bar{3}\bar{6}2$  and  $\bar{3}\bar{6}2$ . We have already shown how the Goldschmidt symbols may be transformed into those of Miller. The reverse may be accomplished by dividing the first two indices by the third, thus 362 in the system of Miller becomes  $\frac{2}{3}3$  according to Goldschmidt.

#### 10. General Properties of Polyhedra.

The smallest number of plane surfaces capable of enclosing space is four. This is crystallographically represented by the tetrahedron. On the tetrahedron there are four faces, four corners and six edges. The cube possesses six faces, eight corners and twelve edges. The relationship between the num-

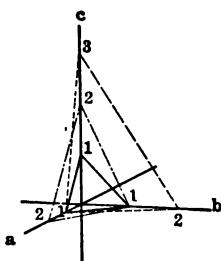


Fig. 17

bers of edges, corners and faces exhibited by these two polyhedra is a general characteristic for all polyhedra and may be stated as follows:

$$\text{Faces} + \text{corners} = \text{edges} + 2$$

When an edge or a corner on a polyhedron is removed by a single plane the process is known as truncation. Figures 18 and 19 show truncation of edges and corners as applied

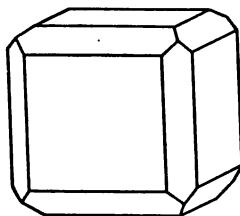


Fig. 18

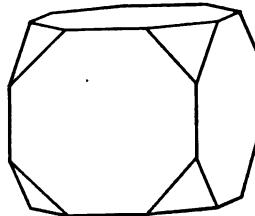


Fig. 19

to the cube. Where the truncating plane makes equal angles with the planes forming the edges or corners the process is known as symmetrical truncation. In all other instances it is unsymmetrical. Bevelment is the removal of an edge by two planes. It may be symmetrical or unsymmetrical. Three or more planes removing a corner give rise to acumination. Figures 20 and 21 show the symmetrical bevelment

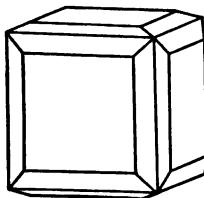


Fig. 20

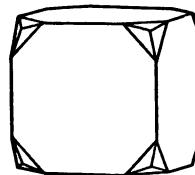


Fig. 21

and acumination as applied to the edges and corners of the cube. The Miller's symbol for a face which symmetrically truncates an edge or a corner is the algebraic sum of the indices of the faces which meet to form the edge or corner concerned, thus, in Figure 18 the symbols of the two cube

faces forming the edge truncated are (100) and (001), while the symbol of the truncating face is (101). Similarly in Figure 19 the symbols of the three cube faces meeting to form the corners truncated are (100), (010) and (001) and the symbol of the truncating face — the octahedron — is (111).

## CHAPTER V

### SYMMETRY OF CRYSTALS

Frequent reference has already been made to the symmetrical arrangement of the points of both stereographic and gnomonic projections. When examining crystals this symmetry shows itself in a slightly different way. If a plane be passed through a cube centre and parallel to a pair of cube faces as shown in Figure 22 it becomes at once apparent

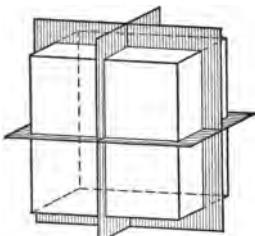


Fig. 22

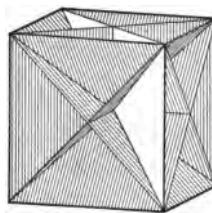


Fig. 23

that for each corner or edge shown on one side of that plane there is a corresponding corner or edge on the opposite side. If any point on the surface of the cube be selected, a normal drawn from that point to this plane and produced an equal distance beyond the plane it will always be found to reach a point similar in situation to the one from which it started. A plane which divides a polyhedron into two right and left parts which are similar to one another according to this test is said to be a plane of symmetry, and the polyhedron is symmetrical with regard to that plane. In the cube there are three planes of symmetry, each passing through the centre and each parallel to a pair of cube faces. There are also six other planes of symmetry in the cube, each plane passing through its central point and through two parallel edges. (Figure 23.) A plane of the class first mentioned divides the cube into two parallelopipeds. A plane of the

second class divides it into two triangular prisms. There are therefore in the case of the cube three planes of one kind and six planes of the other. Crystals being polyhedral forms frequently have their faces so arranged as to be symmetrical about one or more planes.

In most cases the faces which belong to a crystal form are not situated at the same distance from the centre and as a result the crystals do not exhibit the ideal geometrical form and are said to be distorted. Figure 24 shows this

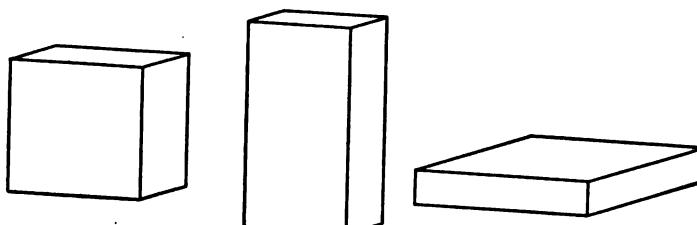


Fig. 24

distortion with reference to the cube. In all such cases the inclinations of the faces in the distorted forms are identical with those of the ideal form. This is in accordance with the law of constancy of interfacial angles. In considering the presence or absence of planes of symmetry in crystals we always have in mind the idealized crystal which would result by giving equal central distance to all those faces belonging to the same *crystal form*. Figure 25 represents an

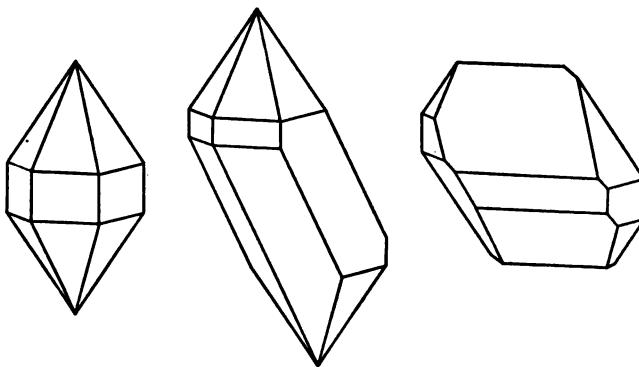


Fig. 25

idealized crystal of the mineral quartz ( $\text{SiO}_2$ ), also crystals of quartz in which distortion results from the several faces of the same crystal form occurring at different distances from the centre.

If a cube be revolved about an axis passing through the central points of two opposite faces it will be observed that after it has been turned through an angle of  $90^\circ$  the edges and corners of the cube occupy the same positions as did other edges and corners before the revolution was begun. The cube when revolved about a line having this direction will present to the observer the same picture at intervals of  $90^\circ$ . There are in the cube three axes of this type. If rotated about an axis passing through two diagonally opposite corners the cube will present to the observer the same picture at intervals of  $120^\circ$  — three times during the revolution. There are four directions of this type in the cube. Lastly, if a cube be revolved about a line passing through the middle points of two opposite and parallel edges it presents to the observer the same picture twice during the revolution. In the cube there are six axes of this type. An axis about which a crystal may be revolved so as to present to the observer the same picture two or more times during the revolution is called an axis of symmetry. In the cube there are three different kinds of axes of symmetry. Those axes which present the same picture four times during a revolution are called tetrad axes, similarly those presenting the same picture two or three times are known as diad and triad axes respectively. These axes of symmetry are shown in Figure 26. Crystals in their idealized form are usually symmetrical about one or more axes. The kinds of axes of symmetry as exhibited in crystals are diad, triad, tetrad and hexad. Pentad symmetry, which is so characteristic of living forms, has never been observed in the case of crystals and indeed is not mathematically possible. The axes of symmetry are sometimes designated as binary, trigonal, tetragonal and hexagonal.

One other element of symmetry is very frequently exhibited by crystals, viz., symmetry about a centre which implies the

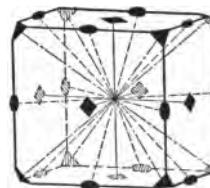


Fig. 26

occurrence on opposite sides of a crystal of parallel pairs of faces. The complete symmetry of the cube may be stated as follows:

- (a) *Axes of symmetry* — six diad, four triad and three tetrad.
- (b) *Planes of symmetry* — six secondary planes, three chief planes.
- (c) *Centre of symmetry present.*

Classes of polyhedra involving only the following elements of symmetry: planes, centre and diad, triad, tetrad and hexad axes. If we attempt to work out mathematically and classify the possible types of crystal symmetry we find that they fall into thirty-two distinct groups known as *classes*. Each one of these classes has a symmetry different from every other. There are for example certain crystals which do not present any element of symmetry — these belong to the hemihedral subdivision of the triclinic system. On the other hand the crystals belonging to the main division of the triclinic system are symmetrical about the centre only. In these two classes the crystals are of a very low degree of symmetry. The crystals of the cubic system presenting the symmetry already indicated for the cube are more highly symmetrical than those of any other group. When first this maximum number of possible crystal *classes* was announced there were certain classes which at that time had no known representative. At the present time representatives are known for all but one or two of the thirty-two classes.

There is an interesting correspondence between the complexity of chemical composition and the degree of symmetry shown by crystals. Most of the substances devoid of symmetry or possessing only a centre of symmetry have very complex chemical formulæ such as strontium bitartrate,  $\text{Sr}(\text{C}_4\text{H}_4\text{O}_6\text{H})_2 \cdot 4 \text{H}_2\text{O}$ , or albite,  $\text{NaAlSi}_3\text{O}_8$ . In contrast to this many of the substances crystallizing in the cubic system and exhibiting a high degree of symmetry are chemically extremely simple — galena,  $\text{PbS}$ , zinc blende,  $\text{ZnS}$  and the chemical elements copper, gold, silver and carbon in the form of diamond. While one cannot fail to observe this correspondence between chemical simplicity and high symmetry

on the one hand and chemical complexity and low symmetry on the other, it is only a general relationship for there are certain substances chemically complex showing high symmetry and others chemically simple showing low symmetry.

**Crystal Systems.** An older classification of crystals and one still generally followed for didactic purposes subdivides all crystals into six systems. Each of these systems contains several classes so that within a system there are two or more classes of crystals each with its own peculiar degree of symmetry. All those crystal classes which belong to one system are closely related to one another: (a) their forms may be referred to the same set of axes; (b) they resemble one another very closely in their physical properties; (c) it is usually possible to derive the crystal forms of the classes showing lower symmetry from forms of that class presenting the highest symmetry for the system by supposing that such forms of lower symmetry have been obtained by an extension of one-half or one-quarter of the faces of some crystal form belonging to the class within the system which shows the highest symmetry; (d) sometimes crystal forms which belong to the highest class of the system occur on crystals in combination with forms which are characteristic of one of the less symmetrical classes of the system, e.g., a crystal belonging to the cubic system may be bounded by the six planes of the cube and the twelve planes of the pyritohedron as shown in Figure 27. Now the pyritohedron is lower in symmetry than

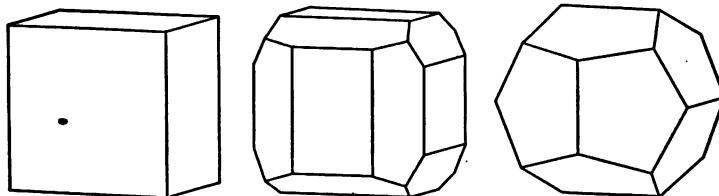


Fig. 27

the cube but the two of them occur together on the same crystal as a combination. The subdivision of the thirty-two classes into six systems is so arranged therefore that all those classes which are placed under any one system are very closely related to one another although not identical in symmetry.

The following are the names of the six systems:

1. Cubic, tesserall, isometric or regular,
2. Tetragonal or quadratic,
3. Hexagonal,
4. Rhombic, orthorhombic or trimetric,
5. Monoclinic, clinorhombic, monosymmetric or oblique,
6. Triclinic, asymmetric or anorthic.

## CHAPTER VI

### PHYSICAL PROPERTIES

The physical properties in crystals vary with the direction in the crystal but any individual crystal will be found to present the same degree of symmetry from the physical point of view which it presents when geometrically considered. As a result of this remarkable correspondence between the physical and geometrical symmetry it is frequently possible by a physical examination of a fragment of a crystal, which does not show any natural crystal surface, to indicate the system to which it belongs. In the study of thin sections of rocks under the microscope the determination of the system for the various mineral fragments is based almost entirely upon the optical properties of the mineral fragment in question. In general it may be stated that directions which are geometrically equivalent in a crystal are also physically equivalent. The cohesion displayed by a crystal varies with the direction. A crystal of rock salt whose common form is the cube splits readily in three directions parallel to the faces of the cube. A cube of fluorite on the other hand is easily cleaved along four planes, each cleavage being parallel to a plane which symmetrically removes a corner of the cube. In the first of these examples it will be observed that the three directions of minimum cohesion are identical with the directions of the three tetrad axes while in fluorite the directions of minimum cohesion coincide with the four triad axes. The particles of which the crystal is composed appear to be so arranged and to possess such cohesive properties as in themselves to be prophetic of both the geometrical form and the physical properties displayed by the crystal.

If we consider crystals with regard to their optical properties we observe that certain of them are singly refracting without regard to the direction in which the light travels.

Such crystals always possess geometrical forms which place them in the cubic system. All other crystals are doubly refracting although there are certain directions within them in which light may travel without suffering double refraction. These directions of single refraction in doubly refracting substances are known as optic axes. Some crystals possess only one optic axis while others possess two. The former are geometrically either hexagonal or tetragonal, the latter rhombic, monoclinic or triclinic.

It is not intended here to pursue the discussion of the physical properties of crystals but only to emphasize the fact that the symmetry displayed in the geometrical forms of crystals corresponds to that revealed by their physical investigation.

## CHAPTER VII

### THEORY OF THE INTERNAL STRUCTURE OF CRYSTALS

Certain marked regularities in crystals have already been indicated and the scientific mind attempts to frame for itself an hypothesis of a wider range so as to bring all these observations into accord with some fundamental conception. The regularities referred to may be grouped under the following headings:

1. Rationality of axial parameters,
2. Symmetry of crystals,
3. Constancy of interfacial angles,
4. Variation of the velocity of light within a crystal with its vibration direction,
5. General purity of crystals.

To account for these regularities an hypothesis has been formed which supposes that crystals are made up of small particles or molecules which are not necessarily identical with the molecules of the chemist. The particles are regularly arranged in lines at equal distances from one another, parallels of such lines form planes while parallel planes of such particles build up the whole solid structure. These crystal particles are held together by mutually attractive forces. During the building of the crystal there are repellent forces which prevent the mingling together of particles of different kinds in the construction of the same crystal. This net work of lines of particles is spoken of as the lattice. For certain classes of crystals the space distances between the particles in the three directions may be identical as in the cubic system. In other systems these distances may be different from one another. The angles formed by the intersection of the main lines of the lattice may be rectangular or oblique, thus giving rise to many different lattice

structures. According to this hypothesis the crystal faces are planes of points.

In a rock such as hornblende granite the arrangement of the crystal grains is such that we know that the chief minerals — quartz, orthoclase and hornblende — were all crystallized at the same time. The fused magma from which these substances crystallized must have been exceedingly complex. The simultaneous assembling of myriads of particles of three different kinds, their orientation and separation so as to form three different complex crystallized substances has a very good parallelism drawn from military life. In a garrison town where are stationed infantry, cavalry and artillery the men at certain hours are allowed to wander with the greatest freedom. At a given signal they assemble the infantry in one place in regular order in series of parallel lines, while the cavalry and artillery each has its peculiar place of meeting and its own order when together. So do the crystal particles of different substances when taking on the crystallized condition gather about certain centres, orient themselves with reference to the particles earlier fixed, and at the same time prevent the intrusion of particles foreign to those already placed.

All substances whose particles are not arranged in a regular manner are said to be amorphous. Substances of this character usually owe their form to rapid solidification caused by sudden cooling or they may be composed of several different chemical individuals. If quartz or garnet be fused and the molten mineral suddenly cooled by pouring it upon an anvil, the quickly cooled substance is amorphous and less dense than the crystallized minerals from which they are formed.

Crystal of quartz S.G. 2.66 — when fused S.G. 2.20

“ “ garnet S.G. 3.63 — “ “ S.G. 2.95

Amorphous products obtained from the fusion of crystallized substances are frequently more easily attacked by corrosives than the same substances in the crystallized condition. The particles are not so closely packed together and as a result occupy a larger space and are more easily detached.

The theory of the internal crystal structure has developed gradually. It was first suggested by Bergmann under the

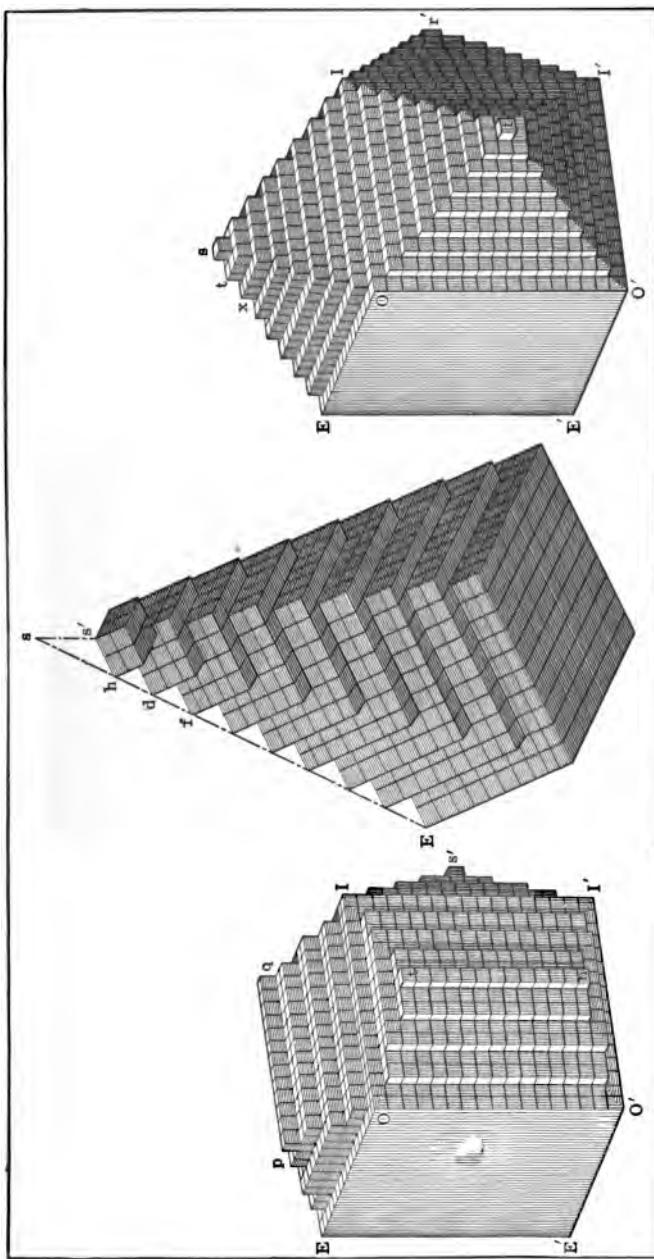


Fig. 28

name of the corpuscular hypothesis. Haüy's observations on the cleavage of certain substances caused him to suggest that crystals are built up of particles of the form enclosed by their most prominent cleavages. Figure 28 is from his "Traité de Mineralogie," 1801, and indicates his application of this hypothesis to the rhombic dodecahedron, pyritohedron and scalenohedron.

When crystals are grown together so that only part of their faces are developed the substance is said to be *crystallized*. In the formation of rocks crystallization frequently begins about a multitude of centres and the crystallized masses afterwards come into contact with one another so that when the whole mass is solidified there are no crystal faces to be observed. The rock is granular and made up of grains in each of which the particles are all of one kind and regularly oriented as shown by an examination of thin sections with a microscope. If the mineral be cleavable the bright reflecting cleavage planes are readily observed on the rock surface. Such a substance is said to be *crystalline*. Masses of halite taken from some salt mines show this structure. Most rock minerals are crystalline; crystallized specimens showing partial enclosure by crystal faces are not common, while the formation of crystals completely enclosed by plane surfaces is rather rare. In all three types the internal structure is identical.

## PART II

### CHAPTER VIII

#### CUBIC SYSTEM

Within each system there are included several *classes* of crystals each of which has its own peculiar degree of symmetry. It is usual when speaking of the degree of symmetry of a system to mention the symmetry which is characteristic of that class possessing the highest degree of symmetry. Within a system this class is said to be holohedral, because from the forms belonging to it the forms of certain other classes may be derived by selecting one-half or one-quarter of the faces. In contrast to the holohedral class the other classes belonging to the same system are said to be hemihedral, tetartohedral or hemimorphic. The exact meaning of these terms will be made plain when it is in order to describe the various subordinate classes belonging to the systems.

**Symmetry of the Cubic System.** The symmetry of the holohedral class of the cubic system has already been indicated in connection with our study of the symmetry of the cube. Crystals belonging to this class exhibit the following elements of symmetry:

- (a) *Planes.* Three chief planes of symmetry, each of which is parallel to two cube faces, and six secondary planes of symmetry, each passing through the centre, and through two diagonally opposite parallel edges of the cube.
- (b) *Axes.* Three tetrad, four triad, six diad.
- (c) *Centre present.*

**Standard Orientation of Cubic Crystals.** Crystallographers have arbitrarily selected for each type of crystal a definite orientation which is always followed for crystals of that particular class. In the cubic system the crystal is so oriented

that its tetrad axes are fore and aft, right and left and vertical.

**Axes of Reference.** The axes of reference for this system consist of three lines at right angles to one another, one fore and aft, one right and left, the third vertical. In crystals of all classes of the cubic system the faces are inclined to the axes in such a manner that the lengths of the intercepts on all three axes are simply and rationally related to one another. This is the only system of crystals in which the axes are intersected by planes in this peculiar manner. Where all three of the axes of reference are intersected by crystal

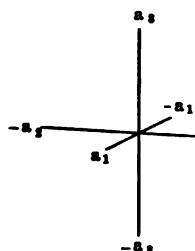


Fig. 29

planes at distances from the centre which are simply and rationally related to one another, it is usual to speak of the axes as equivalent. Figure 29 shows the axial cross for the cubic system.

All three axes are designated by the letter  $a$ , since they are equivalent axes, but to distinguish them from one another, the fore and aft axis is designated by  $a_1$ , the right and left by  $a_2$  and the vertical by  $a_3$ .

**Classification of all Possible Planes Intersecting Three Equivalent Axes at Right Angles to One Another.** A plane may have such a direction with regard to the axes of reference that it intersects only one axis, being parallel to the plane marked out by the other two. Two of the three axes may be intersected by a plane at finite distances, while the third axis is intersected only at infinity. It is possible to further subdivide this type of intersection into two cases:

- (a) The intercepts of the two axes are equal, and
- (b) The intercepts of the two axes are unequal.

There are planes which have such directions as to intersect all three axes, but here again subdivision is possible:

- (a) All three intercepts are equal,
- (b) All three intercepts are unequal,
- (c) Two intercepts are equal, the third greater, and
- (d) Two intercepts are equal, the third less.

In considering therefore a classification of all the possible modes of intersection of planes on three equivalent axes at

right angles to one another we find that there are seven possibilities. Remembering the degree of symmetry which is characteristic of cubic crystals it readily follows that when these axes are intersected by a plane there are a certain number of other planes which are necessarily present in order that the degree of symmetry of the system may be maintained, e.g., if the axes be intersected at equal distances from the centre by a plane it is necessary in accordance with the symmetry that seven other planes should also be present for this particular form. This gives rise to the octahedron, as represented in Figure 30. Where all three axes are inter-

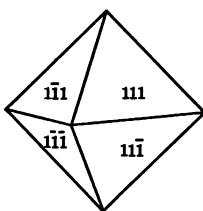


Fig. 30

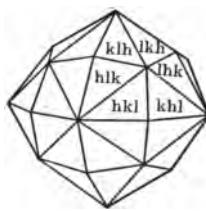


Fig. 31

sected at different distances from the centre, symmetry requires the presence of forty-seven other planes. The form resulting, known as the hexoctahedron, is illustrated in Figure 31. Corresponding to the possibilities referred to there are seven crystal forms which may be indicated as follows:

1. One axis cut — the cube,
2. Two axes cut equally — the rhombic dodecahedron,
3. Two axes intercepted unequally — the tetrahedron or pyramidal cube,
4. Three axes cut equally — the octahedron,
5. Three axes cut unequally — the hexoctahedron,
6. Two axes cut equally, the third greater — the trisoctahedron,
7. Two axes cut equally, the third less — the icositetrahedron.

These are the seven crystal forms characteristic of the cubic system. Certain of them are invariable in their form (1, 2 and 4) while others are more or less variable since where it is merely stated that two or three axes are intersected at

unequal distances from the centre there are a multitude of crystallographic possibilities. In the hexoctahedron the planes may have such directions as to intersect the three axes in various ratios such as the following:

$$\begin{aligned} & 1a_1, 2a_2, 3a_3, \\ & 2a_1, 3a_2, 5a_3, \\ & 4a_1, 5a_2, 6a_3. \end{aligned}$$

These are all hexoctahedra. On diamond crystals the following hexoctahedra are found: (321), (431), (541), (651). Such types as the hexoctahedron are said to be *variable forms*.

**Cube.** The cube is bounded by six planes at right angles to one another meeting to form twelve edges and eight trihedral corners (Figure 24). In the cube of geometry these six faces are all equidistant from the centre and as a result the six planes are squares. In crystals on the other hand these planes frequently are at different distances from the centre and as a result the six surfaces are seldom squares. They are rectangles of varying proportions. The departure from the idealized geometrical form as a result of varying central distances for planes belonging to the same crystal form is called distortion.

The eight sectants into which space is divided by the three chief planes of symmetry, each of which includes two of the axes of reference, are known as octants. Figure 22 indicates this arrangement of the octants with regard to the cubic system. In the formation of the symbol to designate a whole crystal form it has been agreed that the face symbol employed for that purpose shall belong to a face contained in the right front upper octant. Where, as in the case of the cube and the rhombic dodecahedron, several faces appear within the octant, the selection is still further limited by the requirement that the face must be one of those intersecting the fore and aft axis at the shortest distance. Where there are two faces intersecting this axis at equal distances the lower one as they lie in the octant is selected to give the form symbol. By this agreement all crystallographers use the same face symbol for the derivation of the form symbol. The parameter ratios for the front face of the cube are  $1a_1, \infty a_2, \infty a_3$  so that the Miller's symbol for this face would

be represented by 100 which, when enclosed in brackets thus (100), stands as the symbol for the whole crystal form. The minerals pyrite,  $\text{FeS}_2$ , fluorite,  $\text{CaF}_2$ , and galena,  $\text{PbS}$ , frequently crystallize as cubes.

**Rhombic Dodecahedron.** As its name implies, this form is bounded by twelve faces, each of which is a rhombus (Figure 32). These faces meet to form twenty-four edges, six tetrahedral corners marking the points of exit of the tetrad axes of symmetry, and eight trihedral corners at the point of exit of the four triad axes of symmetry. The angles over the edges between the faces are all equal and measure  $60^\circ$  (outer angles). The parameter ratios for the faces of the rhombic dodecahedron are as follows:

$$1a_1, 1a_2, \infty a_3$$

This corresponds to (110) in the index symbols of Miller.

**Octahedron.** This is one of the simplest forms of the cubic system, inasmuch as its faces intersect all three axes at equal distances from the centre. As its name implies it is composed of eight faces intersecting in twelve edges and six tetrahedral corners. In the idealized form where all the faces have the same central distance each face is an equilateral triangle (Figure 30). The parameter ratios for a face of this form are:

$$1a_1, 1a_2, 1a_3,$$

so that the Miller's symbol for the whole form is (111). Examples: Pyrite,  $\text{FeS}_2$ , diamond, alum,  $\text{KAl}(\text{SO}_4)_2 \cdot 12\text{H}_2\text{O}$ .

**Tetrahexahedron or Pyramidal Cube.** This form resembles in appearance a cube on each of whose six faces has been placed a flat, four-sided pyramid (Figure 33). It is bounded by twenty-four faces which meet to form twelve cubic edges which are parallel to the edges of the cube and twenty-four dodecahedral edges which are approximately parallel to the edges of the rhombic dodecahedron. There are six tetrahedral corners marking the points

of exit of the tetrad axis and eight hexahedral corners indicat-

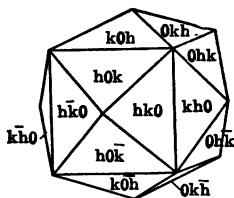


Fig. 32

of exit of the tetrad axis and eight hexahedral corners indicat-

ing the position of the four triad axes. In the idealized form where all the faces possess the same central distance, each of its twenty-four faces is an isosceles triangle. Each of the faces of this form intersects two of the axes of reference at finite distances which are unequal. We may, therefore, anticipate a great variety of tetrahedra, all conforming with the above general description. The two axes intersected may exhibit such ratios as the following:

$$\begin{aligned}1a_1, 2a_2, &\infty a_3, \\1a_1, 3a_2, &\infty a_3, \\2a_1, 3a_2, &\infty a_3.\end{aligned}$$

The Miller's symbols for these three types are (210), (310), (320). On crystals of the mineral fluorite the following tetrahedra have been observed: (610), (510), (920), (410), (310), (520), (730), (210) and (530).

**General Symbols.** In order to write a symbol which will stand for all possible tetrahedra, it is necessary to introduce certain letters, which according to convention vary within certain limits. It has been agreed that the letters  $h$ ,  $k$  and  $l$  shall represent indices with the relative values  $h > k > l$ . Where it is only necessary to use two of these in order to write a symbol,  $h$  and  $k$  are the ones selected. According to this convention all possible tetrahedra may be represented by the general symbol  $(hk0)$ . Whenever we wish to represent by one general symbol all the possible variations of a variable form it is necessary to use the letters  $h$ ,  $k$  and  $l$ .

**Hexoctahedron.** (Figure 31.) This form, as its name suggests, is bounded by six times eight faces—forty-eight in all. On no other crystal form does such a large number of faces occur. The number of faces in the form is due to the high symmetry of the cubic system, and to the fact that its faces are planes which meet all three axes at different intercepts. In the cubic system a crystal face with such a direction as to intercept the three axes unequally, requires the simultaneous occurrence of forty-seven other faces, similarly oriented with regard to the axes. In the idealized form each of the forty-eight faces is a scalene triangle. There are seventy-two edges—twenty-four cubic, twenty-four octahedral and twenty-

four rhombic dodecahedral. These terms are applied to the various edges on account of their lying approximately in line with the characteristic edges of the cube, the octahedron and rhombic dodecahedron. There are corners of three types, six octahedral corners marking the point of exit of the tetrad axes, eight hexahedral corners at the exit of the triad axes and twelve tetrahedral corners indicating the point of exit of the diad axes. The hexoctahedron, like the tetrahedron, is a variable form. Since according to its definition it is only necessary that a face of the hexoctahedron intersect the three axes unequally it is possible to conceive of a large number of ratios for the lengths of the three intercepts. On crystals of diamond, as has already been stated, the following hexoctahedra have been observed: (321), (431), (541), (651). The general symbol representing these and all other possible hexoctahedra is (hkl).

**Trisoctahedron.** Crystals of this form resemble closely the octahedron and may be regarded as an octahedron on each of whose faces a flat, three-sided pyramid has been placed, as illustrated in Figure 34. The trisoctahedron is bounded by twenty-four planes, which in the idealized form appear as isosceles triangles. There are two kinds of edges, the octahedral and the rhombic dodecahedral. Six hexahedral corners and eight trihedral corners mark the points of exit of the tetrad and triad axes respectively. The general symbol for all possible trisoctahedra is (hhk); (221) has been observed on diamond and cuprite, (331) on native silver and cuprite, (661) on pyrite.

**Icositetrahedron.** This form is also known as the leucitohedron and trapezohedron. One of these names is given to it on account of its being the characteristic form for the mineral leucite, while the last is suggested by the geometric outline of the plane. There are twenty-four faces, three in each octant — twenty-four octahedral edges and twenty-four cubic edges, three kinds of corners, six tetrahedral marking the positions of the tetrad axes, twelve tetrahedral at the point of exit of the diad axes and eight trihedral marking the middle point of each octant. Like the last three forms the

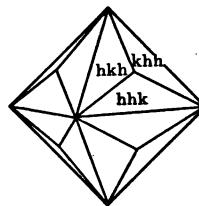


Fig. 34

icositetrahedron is variable and the letters h and k must be utilized in writing a general symbol ( $hkk$ ) for all possible icositetrahedra. The forms (811), (411), (311) and (211) occur on crystals of native gold. Other minerals on whose crystals the icositetrahedron is frequently observed are garnet, native copper and native silver (Figure 35).

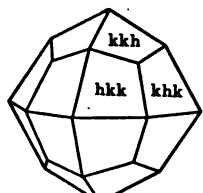


Fig. 35

It might appear at first sight that there would be found on crystals a very great variety of representatives of any one variable form. As a matter of observation, however, we learn that face indices larger than ten are very uncommon in crystal symbols. In the trisoctahedron, for example, as a result of this the actual number of variations for ( $hkk$ ) is not great and on all crystals showing this form measured up to the present time it is probable that not more than a dozen distinct forms have been observed. This limitation is due to the rarity on crystals of such faces as involve indices larger than ten.

**Combinations of Crystal Forms.** Crystals which are completely enclosed by the faces of a single crystal form are very unusual. Much more frequently on the crystal, two or more forms are to be observed, the faces of the one

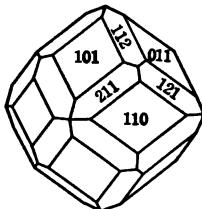


Fig. 36 — Garnet showing the rhombic dodecahedron and the icositetrahedron (211).

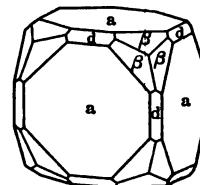


Fig. 37 — Cuprite showing the cube,  $a$ , (100) the rhombic dodecahedron,  $d$ , (110) and the icositetrahedron,  $\beta$ , (322).

form removing the corners or edges of the other. The combination of these different forms is always such that the directions of symmetry of one form are coincident with those of any other form occurring upon that crystal. Figures 19, 20, 36 and 37 illustrate some of the simplest combinations for the cubic system.

**Limiting Forms.** Attention has already been called to the fact that three of the seven crystal types characteristic of the cubic system are invariable. It is not necessary in writing their symbols to make use of the letters  $h$ ,  $k$  and  $l$ . In each of the other four types, however, there is considerable variation. For each of the forms observed a definite symbol can be written, but a formula representing in a general way all possible tetrahedra involves the use of the letters  $h$  and  $k$ . Some of the tetrahedra occur in forms which closely approximate the cube. For these the four-sided pyramids which rest on each cube face are very flat. For such forms the intercept of one axis is very large compared with that of the other, that is,  $h$  is very much greater than  $k$ . Other tetrahedra seem to resemble more closely in outline the rhombic dodecahedron. In them the four-sided pyramid which rests on each of the cube faces is comparatively sharp. This is due to an approximate equality between the lengths of the intercepts on the two axes. Here it may be observed that the extremely flat tetrahedron approaches the cube in form while the steep-faced tetrahedron approximates the rhombic dodecahedron. By writing one above the other the symbols for these forms, thus:

(100)  
(hk0)  
(110)

the relationship of the three forms to one another may be made plain. The ratio between  $h$  and  $k$  is variable. In certain forms  $h$  and  $k$  are very nearly equal as would be the case for the form (760), in this way giving rise to a tetrahedron very close to the rhombic dodecahedron. On the other hand where  $h$  is very much larger than  $k$  we may obtain a symbol such as (910) which approaches very near to the cube symbol (100). We say, therefore, that the tetrahedron is a form varying between the cube and the rhombic dodecahedron. These two are said to be its limiting forms. Similarly the icositetrahedron approaches the octahedron when  $h$  and  $k$  are nearly equal to one another and the cube when  $h$  is much larger than  $k$ . The limiting forms for the trisoctahedron are the rhombic dodecahedron and the

octahedron. In the hexoctahedron (hkl) where all three indices differ in value we have a large number of possible variations, thus:

(a) When l becomes zero we obtain (hk0) — the tetrahedron,

(b) When l increases in value until equal to k we obtain (hkk) — the icositetrahedron,

(c) When k increases so as to be equal to h we obtain the form (hhk) — the trisoctahedron.

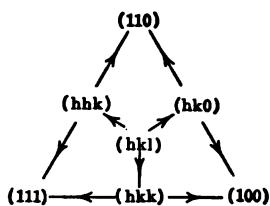


Fig. 38

In the study of limiting forms of the cubic system, therefore, we find that the hexoctahedron reaches its limits in three other variable forms, each of which varies between two fixed limits.

This is expressed diagrammatically in Figure 38.

#### HEMIHEDRAL CLASSES OF THE CUBIC SYSTEM

The name hemihedrism is applied to certain forms within a crystal system which may be derived by the selection of one-half the number of faces present on one of the holohedral forms and by their extension until they intersect in edges and corners. The forms which are characteristic of three of the subordinate classes of the cubic system can be obtained in this way. The fourth or tetartohedral class presents only such forms as might be obtained by a regular selection and extension of one-quarter of the faces belonging to certain forms of the holohedral class. The symmetry of the subordinate classes in any system is always lower than that of the chief or holohedral class. The hemihedral and tetartohedral forms usually occur on crystals in combination with certain of the forms which are characteristic of the holohedral class. Only certain of the forms of the holohedral class are capable of giving rise to hemihedral or tetartohedral forms. The cube for example possesses six planes and it is impossible to select one-quarter of these. The hexoctahedron on the other hand with its forty-eight planes may be extended in quarters and halves so as to give rise to several new forms, hemihedral and tetartohedral in character.

## PLAGIOHEDRAL HEMIHEDRAL CLASS

In the hexoctahedron there are six faces in each octant, none of which is common to two octants. If we select alternately one-half of the faces of the hexoctahedron and extend them to enclose space we find we obtain two new

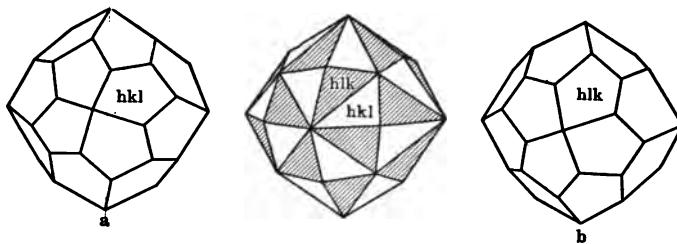


Fig. 39

forms as represented in Figure 39. These new forms are known as pentagonal icositetrahedra.

Each of these forms is bounded by twenty-four planes each of which is irregularly pentagonal. These two forms are similar to one another but by no ingenuity can they be so set up as to present exactly the same picture to the observer. They are related to one another in the same way as the right hand is to the left or as the mirror image is to the picture of the object as viewed directly. Such pairs of forms are said to be enantiomorphous. The symmetry displayed by these plagiobhedral forms is considerably less than that of the holohedral division of the system to which they belong. A careful examination will show that no face is represented by a corresponding parallel and opposite face — the centre of symmetry has disappeared. All the planes of symmetry have vanished while the axes of symmetry characteristic of the holohedral class all remain. The symmetry of the plagiobhedral class is therefore three tetrad axes, four triad axes and six diad axes. These forms have been observed on the mineral cuprite,  $\text{Cu}_2\text{O}$ , where they occur in combination with the cube and the octahedron (Figure 40). It is probable also that the minerals halite,  $\text{NaCl}$ , sylvite,  $\text{KCl}$ , sal-ammoniac,  $\text{NH}_4\text{Cl}$  and cerargyrite,  $\text{AgCl}$ , belong to this class.

The Miller's symbols for hemihedral and tetartohedral

forms are obtained by writing the symbol for a particular face, enclosing it in brackets and placing in front of the bracket a Greek letter which is characteristic for the particular class of hemihedrism. For the plagiobhedral class this letter is  $\gamma$  so that the symbols for the two pentagonal icositetrahedra are  $\gamma(hkl)$  (Figure 39a) and  $\gamma(hlk)$  (Figure 39b). These are known as the left and right pentagonal icositetrahedra respectively.

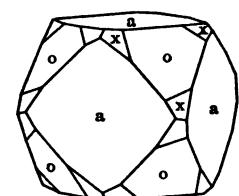


Fig. 40—Cuprite showing the cube,  $a$ , (100); the octahedron,  $o$ , (111); and the pentagonal icositetrahedron,  $x$ ,  $\gamma$  (986), according to Miers.

of the cubic system. Let us note that this method of selection involves:

(a) The extension of one-half the faces within the octant, and

(b) Suppression of the other half.

It is therefore inapplicable to the cube, octahedron, rhombic dodecahedron, trisoctahedron and icositetrahedron, because for all of these forms the number of faces contained within the octant is odd, so that the selection of one-half is not possible. In the tetrahedron there are contained within the octant six faces but it is not possible to secure the suppression of one-half the faces because each face is common to two adjacent octants. The face, therefore, which is suppressed in one octant is reproduced by the extension of the half of it which is selected for extension in the adjacent octant. This is illustrated in Figure 41. As previously mentioned the pentagonal icositetrahedra occur on crystals in combination with various forms characteristic of the holohedral class of the cubic system. The internal molecular arrangement which gives rise to the pentagonal icositetrahedra appears to be able to express itself in such forms as the cube, the octahedron or the tetrahedron. These forms,

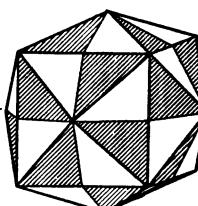


Fig. 41

however, are in certain respects physically distinct from the corresponding forms in the holohedral division. We therefore refer to them as hemihedral cubes, octahedra, etc.

#### PENTAGONAL HEMIHEDRAL CLASS

The characteristic forms belonging to this class of hemihedrism may be obtained by selecting alternately one-half the faces of certain holohedra. While the method of selection of the faces within the octant is the same as for the plagiobhedral class the difference lies in that for the plagiobhedral class the alternation was strictly continued when passing from one octant to the next, while in the pentagonal class

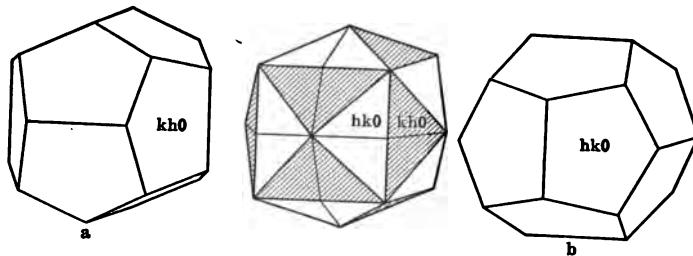


Fig. 42

adjacent faces in contiguous octants are either both extended or both suppressed as is shown in Figure 42.

As a result of this difference in the selection the three chief planes and centre of symmetry are retained in the forms belonging to the pentagonal class of hemihedrism. Only certain holohedra give rise to new hemihedral forms of the pentagonal class. Since one-half the faces within the octant are to be selected only those holohedra which contain within each octant an even number of faces need be examined. Both the hexoctahedron and the tetrahexahedron produce hemihedral forms by this method of selection. The symmetry of the pentagonal forms is represented by four triad axes, three diad axes, three planes of symmetry and the centre. On account of the presence of the centre of symmetry the faces of these forms occur in parallel pairs and this class of hemihedrism is sometimes referred to as parallel-faced. It is also known as pyritohedral on account of pyrite being

one of the commonest minerals to crystallize with this symmetry. Since all three of these names begin with the letter *p* the Greek letter  $\pi$  is placed before the Miller's symbol for the pentagonal forms.

**Diploid.** By the application of the pentagonal method of selection to the hexoctahedron there result two twenty-four faced forms resembling somewhat the icositetrahedra. In each octant there are three four-sided faces. The cubic and octahedral edges are present. There are three kinds of corners, eight trihedral corners marking the middle point of the octants, six tetrahedral corners of one kind and twelve of another, the former marking the points of exit of the diad axes, the latter occupying a position in the planes of symmetry and approximately midway between the axes of symmetry. If on the hexoctahedron the face  $hkl$  be marked as one of the twenty-four to be extended, the form represented in Figure 43b will be obtained. On the other hand if  $h\bar{k}\bar{l}$

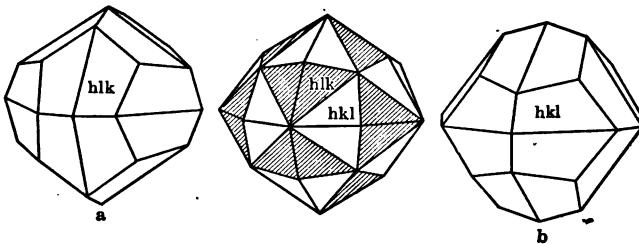


Fig. 43

and the other twenty-three faces which must be selected at the same time be extended, a slightly different form will be obtained, as represented in Figure 43a. These forms are known as diploids. It may be remembered that in the plagiobedral class a pair of forms was obtained by starting from the hexoctahedron. These two were similar to one another but were characterized by a right and left handed arrangement so that it is impossible by any method of orientation to bring the two into a position of absolute parallelism. Such forms are said to be enantiomorphous. The two forms obtained by the application of pentagonal hemihedrism to the hexoctahedron are not so related to one another. Geometrically they are not only similar but identical. If one

of these forms be turned through an angle of  $90^\circ$  about any one of its diad axes it will then be in all respects parallel to the other form. Here the two forms differ from one another only in orientation and are said to be congruent. They are distinguished as positive and negative. The former has the symbol  $\pi(hkl)$ , the latter  $\pi(hlk)$ .

**Pyritohedron.** As a result of the application of this method of selecting one-half the faces to the tetrahedron two new forms are obtained known as pyritohedra or pentagonal dodecahedra (Figure 42); the former name has reference to the frequent occurrence of the mineral pyrite in these forms, while the latter becomes at once apparent by reference to the number and outline of the faces. The pyritohedron is bounded by twelve planes, each approximately pentagonal in outline. There are thirty edges, twenty-four of which radiate in sets of three from the middle of the octant while the other six occupy positions corresponding to the middles of the cube faces. The corners are all trihedral. Eight of them mark the central point of the octant and twelve have positions approximately indicated by the point of exit of the diad axes for the holohedral class. The two forms, designated as positive and negative, bear the same relation to one another as the corresponding diploids. Symbols:

1. Positive —  $\pi(hk0)$
2. Negative —  $\pi(kh0)$

The diploids and pentagonal dodecahedra are commonly found on the minerals of the pyrite and alum groups. The following are a few of the commoner diploids observed on pyrite:  $\pi(321)$ ,  $\pi(432)$ ,  $\pi(751)$ ,  $\pi(531)$  and  $\pi(532)$ . On the same mineral the pyritohedra  $\pi(410)$ ,  $\pi(310)$ ,  $\pi(520)$  and  $\pi(210)$  are frequent. There is an interesting geometrical form bounded by twelve regular pentagons. In many respects it is geometrically closely related to certain pyritohedra, but no pyritohedron exactly of this form has ever been observed. If such a form were referred to three equivalent axes at right angles to one another its indices would be  $2. \left( \frac{1 + \sqrt{5}}{2} \right). 0$ , which would be in conflict with the law of the rationality of axial parameters, which permits only such

faces on crystals as give indices simply and rationally related to one another.

The application of this method of selection to the other five holohedral forms would not produce types geometrically different. The diploids and pyritohedra occur in combination with one another and also in combination with these five forms.

When this occurs we regard the latter as being physically hemihedral although geometrically identical with the corresponding holohedral forms. The mineral pyrite frequently occurs in cubes which are striated in such a manner that the striae are parallel to the cube edges

and the sets of striations on adjacent faces are at right angles to one another (Figure 44). This immediately informs us that the lines representing the diagonals of the cube face are not lines of symmetry as they are in the case of an ordinary cube. A cube so striated is symmetrical about only three planes, each of which is parallel to a pair of cube faces. If a crystal be immersed in some solvent or corrosive and withdrawn after a very small amount of its material has been removed, it will usually be found that its faces are marked by a number of corrosion pits. These etching figures or corrosion pits indicate a certain degree of symmetry and on any one crystal face their corresponding directions are parallel. A cube of sylvite corroded in this way gives rise to four-sided pits arranged as indicated in Figure 45.

While these little pits are symmetrical about certain planes these planes do not coincide in direction with the geometrical planes of symmetry characteristic of the cube. In this way we are able to determine that the cube of sylvite is not physically in conformity with the symmetry of the holohedral division of the cubic system. This method of determining the physical symmetry of crystals will be considered in detail later. It is consequently to be expected that crystals which exhibit the characteristic hemihedral forms of a particular

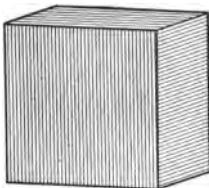


Fig. 44

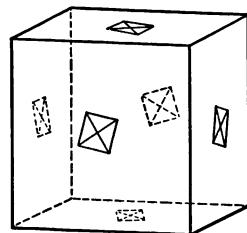


Fig. 45

class will also exhibit those forms of the holohedral class which do not give geometrically new types by the application of this method of selection. Figures 46 and 47 exhibit com-

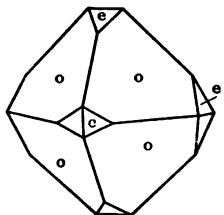


Fig. 46 — Pyrite showing the octahedron,  $o$ , (111) and the pyritohedron,  $e$ ,  $\pi$ (210).

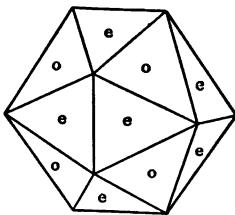


Fig. 47 — Pyrite exhibiting the octahedron and pyritohedron in combination in such a development as to simulate the geometrical icosihedron.

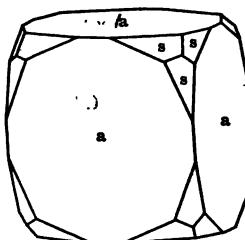


Fig. 48 — Pyrite showing the combination of the cube,  $a$ , (100) and the diploid,  $s$ ,  $\pi$ (321).

bination of the octahedron and pyritohedron. In the latter combination there is a close approximation to the geometrical icosihedron which is bounded by twenty equilateral triangles.

Figure 48 represents the cube in combination with the diploid. Figure 2 shows a very common combination of the cube and pyritohedron.

#### TETRAHEDRAL HEMIHEDRAL CLASS

The crystal forms characteristic of this class may be simply derived by extending the faces of the holohedral forms contained in four alternate octants. All those holohedra whose faces are confined to a single octant give new hemihedral forms by this method. The holohedral forms whose faces occur in only a single octant are the following: octahedron, hexoctahedron, trisoctahedron and icositetrahedron. The forms belonging to this class are characterized by six planes of symmetry, three diad axes and four triad axes. As they are not symmetrical about a centre the faces do not occur in parallel pairs. The letter  $\kappa$  is the characteristic for this class.

**Tetrahedron.** In the octahedron only one face is contained within the octant. The extension of one-half of the

octants, i.e., one-half of the faces of the octahedron is illustrated in Figure 49. There are therefore two congruent

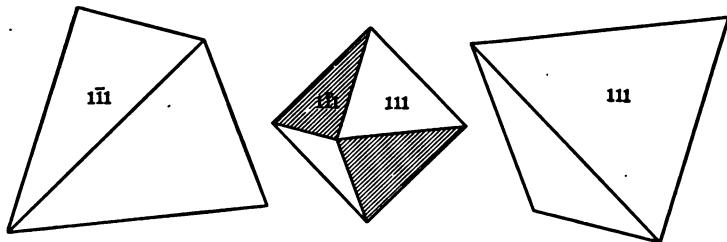


Fig. 49

tetrahedra, one of which results by the extension of the faces which are suppressed in the first instance. These forms are positive and negative with the following symbols:

1. Positive —  $\kappa(111)$
2. Negative —  $\kappa(\bar{1}\bar{1}\bar{1})$

The tetrahedron, as its name implies, is bounded by four planes, each face in the idealized form being an equilateral triangle. There are four trihedral corners and six edges.

**Trigonal Tristetrahedron.** Two trigonal tristetrahedra may be obtained by the application of this method of selection to the icositetrahedron. In outline they resemble a tetrahedron on each of whose faces a flat, three-sided pyramid has been placed in the manner indicated in Figure 50. The trigonal

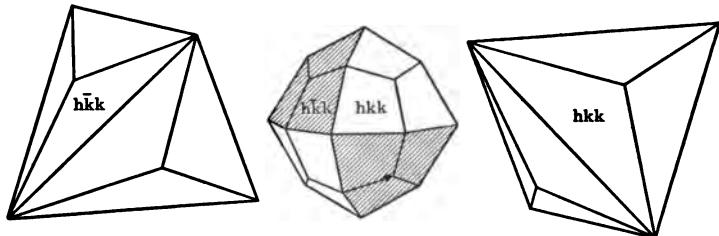


Fig. 50

tristetrahedron is enclosed by twelve faces, each of which is an isosceles triangle. As the icositetrahedron is a variable form so is the trigonal tristetrahedron. The forms are positive and negative and possess the following symbols:

1. Positive —  $\kappa(hkk)$
2. Negative —  $\kappa(h\bar{k}k)$

**Tetragonal Tristetrahedron.** The same method applied to the trisoctahedron gives rise to the forms shown in Figure 51.

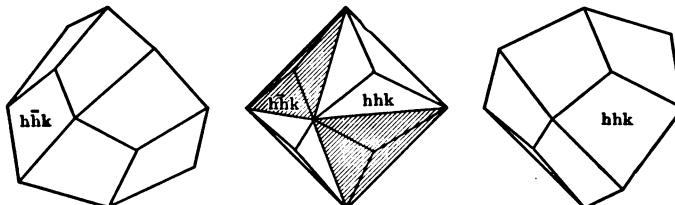


Fig. 51

They resemble the tetrahedron in outline, but the flat, three-sided pyramid is turned in a different direction from that characteristic of the last form. Each of the twelve faces is deltoid resembling in outline the faces of the icositetrahedron. These congruent forms are positive and negative with the following symbols:

1. Positive —  $\kappa(hhk)$
2. Negative —  $\kappa(h\bar{h}k)$

**Hextetrahedron.** By applying the tetrahedral method of selection to the hexoctahedron two new forms result which bear to one another the relationship which characterized the three pairs of the previous forms (Figure 52). In out-

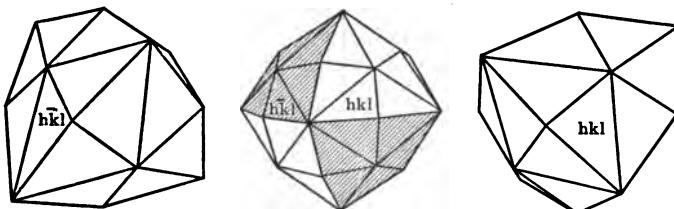


Fig. 52

line the hextetrahedron suggests a tetrahedron on each of whose faces has been placed a flat, six-sided pyramid. Each of the twenty-four faces is a scalene triangle. There are three kinds of corners, four hexahedral marking the central point of the tetrahedral face, four hexahedral at the points corre-

sponding to the corners of the tetrahedron and six tetrahedral corners. They are variable forms and being positive and negative have the following symbols:

1. Positive —  $\kappa(hkl)$
2. Negative —  $\kappa(h\bar{k}l)$

The various new forms resulting from the tetrahedral method of selecting one-half the faces of the holohedra occur in congruent pairs which are geometrically identical and differ from one another only in orientation. These forms occur in combination with one another and with those holohedra of the cubic system which do not give rise to geometrically new types as a result of this method of hemihedral selection. Combinations of hemihedral and holohedral types are shown in Figure 53 indicating simple combination

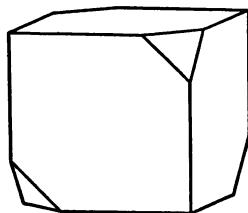


Fig. 53

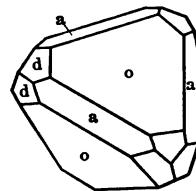


Fig. 54 — The cube,  $a$ , (100); the rhombic dodecahedron,  $d$ , (110) and the tetrahedron,  $o$ ,  $\kappa(111)$ .

of the cube and tetrahedron, and in Figure 54 exhibiting the combination of the forms on the last figure with the addition of the rhombic dodecahedron. Sphalerite, tetrahedrite and diamond are among the best known minerals crystallizing in this class.

#### TETARTOHEDRAL CLASS

There are certain crystal forms which contain one-quarter of the faces of some one of the holohedral forms of the cubic system and which may be regarded as derived by the simultaneous application of any two of the above described hemihedral methods.

It might be thought at first glance that a great variety of tetartohedral forms would result by this method, but a care-

ful examination indicates that it is only the hexoctahedron which gives new forms as a result of the simultaneous application of two hemihedral methods. Indeed, geometrically the same forms are obtained regardless of which pair of hemihedral methods is applied. The surviving faces of the hexoctahedron and the method of selection are indicated in Figures 55 and 56. Three faces survive in alternate octants so that

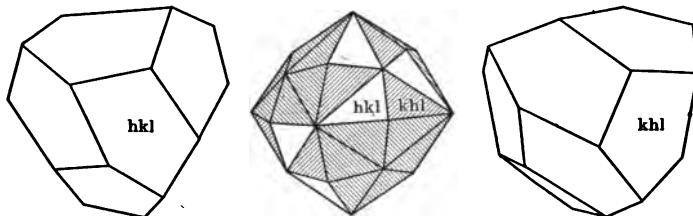


Fig. 55

the new tetartohedral form resembles in a way the tetrahedron each of whose faces is covered by a low, three-sided pyramid. The new form is bounded by twelve planes, each irregularly five-sided and is known as the tetrahedral pentagonal dodecahedron. As indicated in the above figures two enantio-

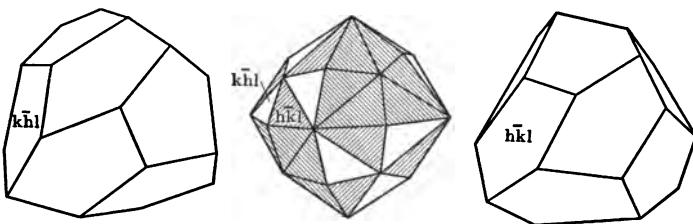


Fig. 56

morphous forms are obtained when three of the surviving faces belong to the front, right upper octant. There is also a pair of enantiomorphous forms resulting when three of the faces to be extended occur in the left, front upper octant. Theoretically therefore there are four tetrahedral pentagonal dodecahedra. Each form of the pair first mentioned is congruent with a form of the second pair. While therefore we have crystallographically four forms, geometrically there are only two. They are designated as follows:

1. Positive right-handed —  $\kappa\gamma(khl)$
2. Negative right-handed —  $\kappa\gamma(hkl)$
3. Positive left-handed —  $\kappa\gamma(hkl)$
4. Negative left-handed —  $\kappa\gamma(k\bar{h}\bar{l})$

Forms 1 and 2 are congruent. The same relationship holds between 3 and 4.

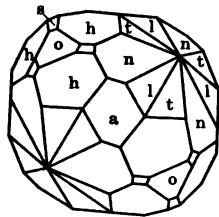


Fig. 57—Barium nitrate showing the cube,  $a$ , (100); the tetrahedron,  $o$ ,  $\kappa(1\bar{1}\bar{1})$ ; the trigonal tristetrahedra,  $l$ ,  $\kappa(311)$  and  $s$ ,  $\kappa(2\bar{1}\bar{1})$ ; and the tetrahedral pentagonal dodecahedra,  $t$ ,  $\kappa\gamma(214)$ ,  $h$ ,  $\kappa\gamma(2\bar{1}\bar{4})$  and  $n$ ,  $\kappa\gamma(351)$ .

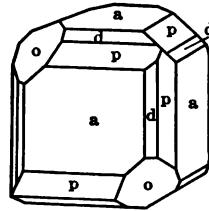


Fig. 58—Sodium chlorate showing the cube,  $a$ , (100); the tetrahedron,  $o$ ,  $\kappa(1\bar{1}\bar{1})$ ; the rhombic dodecahedron,  $d$ , (110) and the pentagonal dodecahedron,  $p$ ,  $\kappa(210)$ .

The tetrartohedral class in the cubic system is exhibited by only a small number of crystallized substances, such as

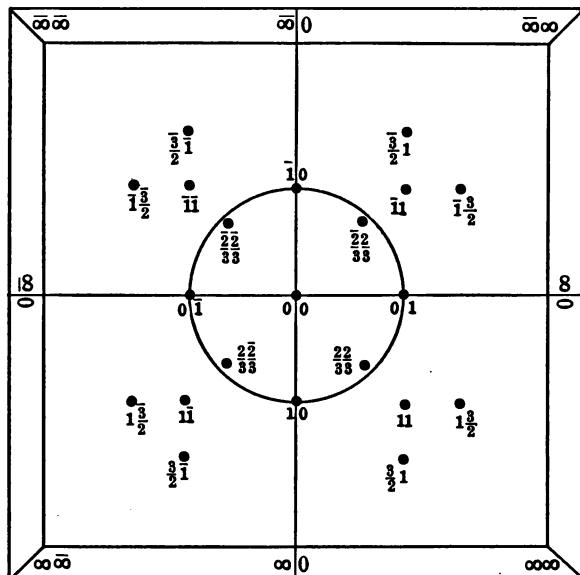


Fig. 59

sodium chlorate, sodium bromate, and the nitrates of lead, barium and strontium. Figure 57 presents a very complex crystal of barium nitrate, exhibiting tetartohedral faces. Crystals on which tetartohedral faces do not occur may be shown to belong to this class of symmetry by the simultaneous occurrence on them of hemihedral forms belonging to different classes, e.g., the simultaneous occurrence on a crystal of tetrahedra and pyritohedra. Figure 58 shows tetartoherdrism of this class for a crystal of sodium chlorate.

**Projections.** Figures 59 and 60 show the character of the gnomonic and stereographic projections respectively for the crystal of galena represented in Figure 61. From the gno-

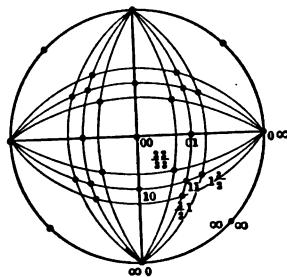


Fig. 60

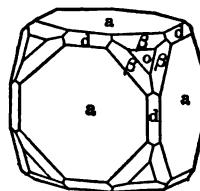


Fig. 61 — Galena showing the cube,  $a$ , (100); the octahedron,  $o$ , (111); the rhombic dodecahedron,  $d$ , (110) and the icositetrahedron,  $\beta$ , (322).

monic projection symbols for the various faces may be readily determined. As has already been pointed out for the cubic system the three axes  $a_1$ ,  $a_2$ ,  $a_3$  being equivalent the polar elements  $p_0$ ,  $q_0$ , and  $r_0$  are all identical. In connection with the cubic system, therefore, the principal problem is the determination of the symbols for the faces present upon the crystal.

## CHAPTER IX

### TETRAGONAL SYSTEM

Within the tetragonal system are contained seven *classes* of crystals, each differing from the others in symmetry. These various classes are closely related to one another in the following ways:

1. They are usually characterized by the presence of a single tetrad axis of symmetry.
2. The faces occurring on crystals of the several classes can always be referred to an axial cross, consisting of three axes at right angles to one another, two of them equivalent, the third different.
3. As in the cubic system there are here certain holohedral forms which by hemihedral and tetartohedral selection do not give rise to new geometrical forms. The forms peculiar to and characteristic of each class frequently occur in combination with these holohedra.

When the tetragonal system is mentioned the reference is to the holohedral class except where special care is taken to indicate one of the classes of a lower degree of symmetry.

#### HOLOHEDRAL CLASS

**Symmetry.** Crystals belonging to this class show the following symmetry:

1. *Axes.* One tetrad; two diad at right angles to one another and to the tetrad axis; two diad axes lying in the same plane as the previous pair of axes and bisecting the angles between them.
2. *Planes.* One plane of symmetry known as the chief plane at right angles to the tetrad axis, and four planes of symmetry each containing the tetrad axis and one of the four diad axes above mentioned.
3. *Centre present.*

**Standard Orientation.** It is customary to orient crystals of the tetragonal system in such a manner that the tetrad axis of symmetry is vertical and so that two diad axes are respectively fore and aft and right and left. Since in this system there are four diad axes lying in the horizontal plane there are two possible methods of orientation for each tetragonal crystal. When, however, a crystal of the tetragonal system is first measured, the crystallographer selects one pair of diad axes rather than the other to have the fore and aft and right and left positions, and henceforth the orientation of crystals of that particular substance is fixed.

**Axes of Reference.** In the tetragonal system there are three axes of reference, one vertical, which coincides with the tetrad axis and a pair of equivalent axes, one fore and aft, the other right and left, coinciding in direction with two diad axes of symmetry. These two lateral axes are spoken of as secondary axes of symmetry while the two others lying in the same plane and midway between them are known as intermediate axes of symmetry. The planes of symmetry containing the vertical and lateral axes of reference are known as secondary planes, while the other two planes of symmetry are referred to as intermediate planes. Any crystal face which intersects all three axes of reference gives intercepts on the lateral axes simply and rationally related to one another. Either these two axes are intercepted at the same distance from the centre or the one is cut at a finite distance, the other at infinity or lastly, the lengths of the intercepts of the two axes are related to one another in some simple ratio such as 2:1, 2:3, or 3:4. As a result these two axes are said to be equivalent and are designated  $a_1$ ,  $a_2$ , as was the case in the cubic system. In the tetragonal system the length of the intercept on the vertical axis is always irrationally related to the length of the intercept on a lateral axis. The vertical axis is therefore not equivalent to the horizontal and in the axial cross is designated by the letter  $c$ . These relationships are exhibited in Figure 62.

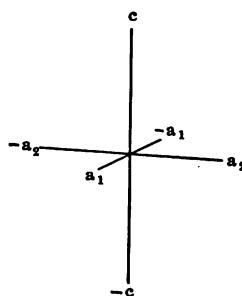


Fig. 62

**Classification of All Planes with Reference to Their Intersection of the Tetragonal Axial Cross.** We can simply divide all possible planes on tetragonal crystals into two classes. Some have such a direction that they do not intersect the vertical axis because there lies in the plane a line which is parallel to this axis. All other planes intersect the vertical axis. Planes of the former class may be subdivided as follows:

(a) The plane intersects both lateral axes

- (1) Equally,
- (2) Unequally.

(b) The plane intersects only one lateral axis.

Planes of the second class may be subdivided thus:

(a) One lateral axis cut at a finite distance,

(b) Two lateral axes cut at finite distances

- (1) Equally,
- (2) Unequally.

(c) Both lateral axes are intersected at infinity.

With regard, therefore, to the way in which planes intersect the tetragonal axial cross there are seven types, giving rise to distinct crystal forms. The seven types, with the names for the tetragonal system are as follows:

1. Vertical axis cut — lateral axes cut equally — pyramid of the first order,
2. Vertical axis cut — lateral axes cut unequally — ditetragonal pyramid,
3. Vertical axis cut — only one lateral axis intersected — pyramid of the second order,
4. Vertical axis cut — lateral axes not cut — basal pinacoid,
5. Vertical axis not cut — lateral axes cut equally — prism of the first order,
6. Vertical axis not cut — lateral axes cut unequally — ditetragonal prism,
7. Vertical axis not cut — only one lateral axis cut — prism of the second order.

**Pyramid of the First Order.** This form, as may be observed from the above sketch, is bounded by planes which intersect the two lateral axes with equal intercepts and the vertical axis with an intercept which is irrationally related to the intercept on the lateral axes. The symmetry of this

system requires that when one such plane appears seven others appear on the crystal at the same time. (Figure 63.)

Such a set of faces is known as a crystal form. There are according to the definition many ways of intersecting the lateral axes equally and the vertical axis at a distance irrationally related to the lateral distances. The one who first describes the crystal of a tetragonal substance arbitrarily selects some pyramid of the first order, usually one represented by large faces upon the crystal and designates it as the *unit pyramid* of the first order. When this has been done the ratio between the lengths of the intercepts on the vertical and lateral axes for a plane of the unit pyramid becomes a characteristic of the substance in question. The length of the vertical intercept for this unit form is indicated by the letter  $c$ , that of the lateral axes by the letter  $a$ . The ratio for these two unit lengths is expressed as follows for the tetragonal mineral zircon:

$$c:a :: .640373:1$$

The vertical intercept is expressed in terms of the lateral intercept,  $a$  being by agreement equal to 1. Sometimes this is expressed still more briefly, thus:

$$c = .640373$$

There are certain pyramids of the first order whose planes are steeper and have an intercept on the vertical axis greater than unity, while others are flatter and possess intercepts on the lateral axes greater than unity. The symbol for the unit pyramid or *ground form* is (111). The steeper pyramids may be symbolized as (hhk) and the flat pyramids as (khh).

The pyramid of the first order is bounded by eight planes each of which is an isosceles triangle. There are four basal edges and eight polar edges. The corners are all tetrahedral, two of them polar and four basal. In a general way these pyramids resemble the octahedron, except that the faces are always isosceles triangles and not equilateral as in the case of the octahedron.

**Ditetragonal Pyramid.** This form results from the intersection of the two lateral axes unequally and the vertical

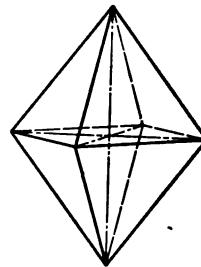


Fig. 63

axis at a distance irrationally related to the lengths of the lateral axes. The symmetry of the tetragonal system requires that such planes should appear in sets of sixteen. The ditetragonal pyramid is bounded by sixteen planes, each of which is a scalene triangle. (Figure 64.) There are eight basal edges and two sets of polar edges — eight in each set.

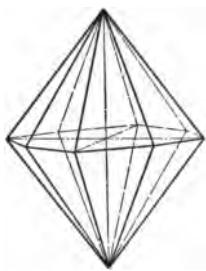


Fig. 64

The corners are of three kinds, two octahedral corners, marking the highest and lowest points of the crystal, four tetrahedral corners at the points of exit of the lateral axes of reference and four tetrahedral corners at the points of exit of the intermediate axes of symmetry. There are many different ditetragonal pyramids, some flatter and some steeper, varying with the relative length of the intercept on the vertical axis. The ratio between the lengths of the intercepts on the

lateral axes may also vary, such values as 2: 1, 3: 2, 4: 3 being among the commonest. It is not difficult to determine the symbol for any particular face since the plane may be supposed to be moved parallel to itself so as to intersect one of the lateral axes at the unit distance, and the other lateral at greater than unit distance, such as two, three or four times. When this occurs the length of the intercept on the vertical axis will be found to be simply related to the unit length on that axis. Thus the parameter ratios on the three axes may be as follows:

$$1a_1: 2a_2: 1c, \text{ or } 1a_1: 3a_2: 2c.$$

From these parameter ratios the Miller's symbols may be obtained as follows:

From  $1a_1: 2a_2: 1c$  the reciprocals  $\frac{1}{1}: \frac{2}{1}: \frac{1}{1}$  are derived.

When these are simplified we obtain the Miller's symbol (212). The general symbol for all ditetragonal pyramids is (hkl).

**Pyramid of the Second Order.** The faces of this form intersect the vertical and one lateral axis, being parallel to the second lateral axis. As a result of the symmetry of the system the crystal form is composed of eight faces, each of which is an isosceles triangle. Geometrically these pyramids are similar to the pyramids of the first order. They are different however in orientation. (Figure 65.)

According to the standard orientation for the tetragonal system, the pyramid of the second order presents a face

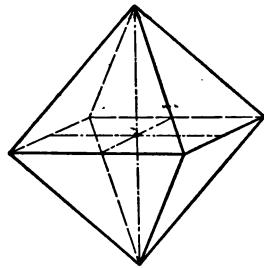


Fig. 65

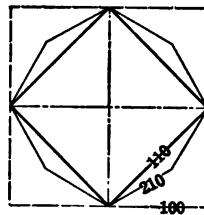


Fig. 66

toward the observer, while its basal edges are right and left and fore and aft. The relative orientation of the three tetragonal pyramids is indicated in Figure 66, while the com-

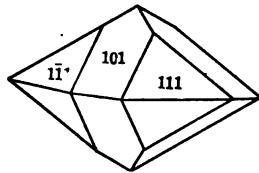


Fig. 67

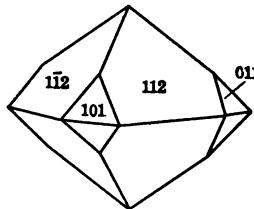


Fig. 68

bination of two orders of pyramids on cassiterite is illustrated in Figures 67 and 68. The general symbol for all pyramids of the second order is  $(h0l)$  — for the unit pyramid  $(101)$ .

**Basal Pinacoid.** The faces of the basal pinacoid are planes which intersect the vertical axis at a finite distance but are parallel to the plane containing the lateral axes. As this system is characterized by the presence of a centre of symmetry there are two such planes comprised in the crystal form. They do not meet to form edges and consequently do not enclose space. Such a crystal form is known as an *open* form in contrast to those already studied

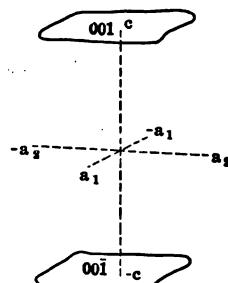


Fig. 69

which are known as *closed* forms. (Figure 69.) The faces of the cube, the octahedron or of the tetragonal pyramids meet in edges and corners and enclose space. Four is the smallest number

of planes which can enclose space, as illustrated by the tetrahedron. Open forms never occur on crystals alone but must necessarily appear in combination with other forms. Figure 70 indicates the combination of the basal pinacoid with the pyramid of the first order. Symbol (001).

**Prism of the First Order.** Planes of this form intersect the lateral axes at equal distances, the vertical axis at infinity. The symmetry of the system demands the presence of four such faces which meet in four edges parallel to the vertical axis. The prism of the first order is an open form, square in cross-section. It, like the basal pinacoid, only occurs on crystals in combination with other forms. Symbol (110). Figure 71 shows this prism in combination with the basal pinacoid.

**Prism of the Second Order.** Here only one lateral axis is intersected by the plane and the whole form is made up of four planes meeting in edges parallel to the vertical axis. The angles between adjoining planes are  $90^\circ$ . This prism is related to the prism of the first order in the same way as the pyramid of the second order is related to the pyramid of the first order. Its symbol is (100). Figure 72 represents the prism of the second order combined with the pyramid of the first order.

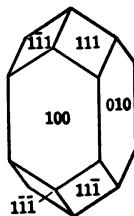


Fig. 70

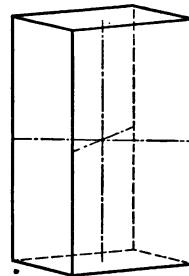


Fig. 71

**Ditetragonal Prism.** Each face of this form intersects the two lateral axes at different distances and in simple ratio, such as 2:1, 3:2 or 4:3. The form is enclosed by eight planes intersecting in edges parallel to the vertical axis. Its cross-section is eight-sided as shown in Figure 66. A ditetragonal prism whose cross-section would be a regular octagon would involve irrationality of the axial parameters, and consequently does not occur

on tetragonal crystals. The ditetragonal prism is a variable form. There are numerous possibilities, since it is only necessary that the ratios of the lateral axes should be simply related to one another. Its general symbol is  $(hk0)$ . Figure 73 exhibits this prism in combination with the basal pinacoid. On the tetragonal mineral rutile the following ditetragonal prisms have been observed:  $(430)$ ,  $(320)$ ,  $(530)$ ,  $(210)$ ,  $(940)$ ,  $(310)$ ,  $(410)$ ,  $(710)$  and  $(810)$ .

**Calculation of the Ratio  $c:a$ .** This may be most simply derived from the inner basal angle between the faces (101) and (101) of the pyramid of the second order. The diagram (Figure 74) illustrates the intersection of such a pyramid by a plane passing through the fore and aft and vertical axes.

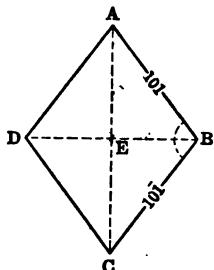


Fig. 74 — Section of the unit pyramid of the second order (101) passing through the vertical axis  $CA$  and the axis  $a_1, BD$ . From this diagram it is apparent that  $c:a : AE : BE$  and that  $c = \tan \angle ABE$ .

cal axes. From this diagram it is readily apparent that the tangent of one-half basal angle equals  $c$ ,  $a$  being unity.

$\angle ABC$  = Basal angle of pyramid of second order.

AE and EB the unit intercepts on the c and a axes respectively.

$$\angle ABE = \frac{1}{2} \text{ basal angle},$$

$$\frac{c}{a} = \frac{AE}{EB} = \tan. \angle ABE = \tan. \frac{1}{2} \text{ basal angle},$$

$\therefore c = \tan. \frac{1}{2} \text{ basal angle.}$

This ratio may be determined from various other interfacial angles. In most other cases it is necessary to employ spherical trigonometry, with which some readers are possibly not familiar:

that  $c = \tan Z ABE$ . The ratio  $c:a$  is a constant which is characteristic for each tetragonal substance. It is identical with the pace length  $p_0$  obtained from the gnomonic projection. For the tetragonal system the constants are thus related:

$$p_0 = c.$$

The holohedral class of the tetragonal system is represented

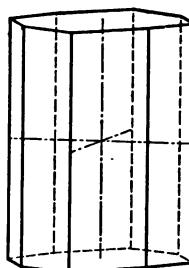


Fig. 73

by the minerals rutile, vesuvianite and zircon, which are shown in Figures 75, 76 and 77.

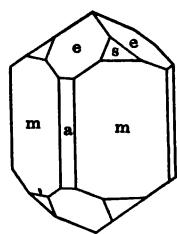


Fig. 75—Rutile showing the prism of the first order,  $m$ , (110); the prism of the second order,  $a$ , (100); the pyramid of the first order,  $s$ , (111) and the pyramid of the second order,  $e$ , (101).

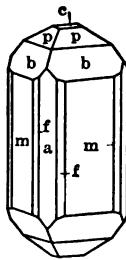


Fig. 76—Vesuvianite showing the prism of the first order,  $m$ , (110); the prism of the second order,  $a$ , (100); the ditetragonal prism,  $f$ , (210); the basal pinacoid,  $c$ , (001) and the pyramids of the first order,  $b$ , (221) and,  $p$ , (111).

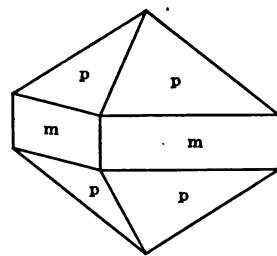


Fig. 77—Zircon showing the prism of the first order,  $m$ , (110) and the pyramid of the first order,  $p$ , (111).

**Limiting Forms in the Tetragonal System.** There are in this system pyramids of three types, the first order, the second order and the ditetragonal pyramids. Of each of these types there are numerous forms, some flatter, others steeper. As a pyramid becomes steeper the two faces which meet to form a basal edge approach more nearly to a position of parallelism. We may therefore regard the prism as an infinitely steep pyramid of the same order. On the other hand the flattening of the pyramid causes the various faces which intersect the same end of the vertical axis to approach the parallel position. When this occurs the basal pinacoid is the resulting form. The pyramid, therefore, of whatever class, may be regarded as variable, swinging between its limits — a prism and the basal pinacoid. So far we have merely considered the variations of the intercept on the vertical axis. There are two types belonging to the tetragonal system which have variable intercepts on the lateral axes — the ditetragonal pyramid and the ditetragonal prism. The general symbol for the ditetragonal prism is  $(hk0)$ . Where  $k$  approaches  $h$  in value we obtain a ditetrago-

nal prism which closely simulates the prism of the first order (110). Where  $k$  becomes so small as to approach 0 a type of ditetragonal prism results approaching closely the prism of the second order. This may be illustrated by the following series of symbols for the various prisms:

(110) — prism of the first order,

(870)

{ (210) — ditetragonal prisms,

(910)

(100) — prism of the second order.

The same relationship maintains between the ditetragonal pyramid and the other pyramids. The limitations for the various forms of the tetragonal system are diagrammatically shown in Figure 78.

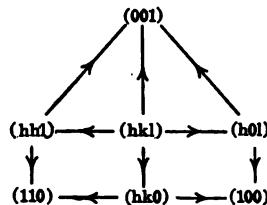


Fig. 78

#### TETRAGONAL CLASSES OF LOWER SYMMETRY

Belonging to the tetragonal system there are in all seven classes. The one already described, commonly known as the holohedral class, presents the highest symmetry. From forms of this class, by selecting one-half or one-quarter of the faces in a regular way, new forms of lower symmetry are obtained. These are the characteristic forms for the various classes of lower symmetry. In this respect the tetragonal and cubic systems are analogous.

#### TRAPEZOHEDRAL HEMIHEDRAL CLASS

This class of hemihedrism resembles in many respects the plagiohedral of the cubic system. Its forms are derived by the selection and extension of one-half the faces within the octant and the simultaneous suppression of the other half. Forms geometrically new can only result under these restrictions when this method is applied to holohedra whose faces do not extend from one octant into the next and where the number of faces within the octant is even. There is only one tetragonal holohedron which fulfils these conditions, viz., the ditetragonal pyramid. Figure 79 illustrates the selection of one-half the faces of this pyramid according to the trapezohedral method. Two enantiomorphous forms

result, one when the face  $hkl$  and the other seven which belong to this set are extended, and a second when the face  $khl$  and its set are extended. These forms, known as tetragonal trapezohedra, are related in the same way as are the two pentagonal icositetrahedra previously described. They are devoid of centre and planes of symmetry but retain the same axes of symmetry as were characteristic for the holohedral class — one tetrad axis, and two pairs of diad axes lying in a plane at right angles to the tetrad axis. These forms are distinguished from one another as right and left. The letter

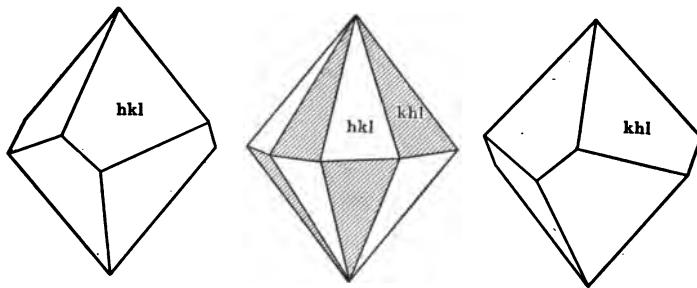


Fig. 79

$\tau$  placed before the Miller's symbol indicates this class of hemihedrism. The symbols for these two forms are therefore:

1. Left,  $\tau(hkl)$
2. Right,  $\tau(khl)$

These peculiar trapezohedral forms have been observed on nickel sulphate,  $\text{NiSO}_4 + 6\text{H}_2\text{O}$ , and on crystals of certain complex organic substances, such as sulphate of strychnine. They occur in combination with the other six forms of the tetragonal system which do not give rise to new forms by this method of selection. The internal molecular arrangement which finds expression in the trapezohedral forms is also capable of building up forms which are geometrically identical with certain of the tetragonal holohedra. When this occurs there is always a physical difference between the holohedral prism, pyramid or pinacoid, and the geometrically identical trapezohedral prism, pyramid or pinacoid. This physical difference may be revealed by means of etching figures, or still more simply, by an optical examination. If

a thin plate be cut parallel to the basal pinacoid and examined in polarized light it will be found that such plates cut from trapezohedral crystals, even though they do not show the characteristic trapezohedral forms, turn the plane of polarization to the right or to the left. In this way we can distinguish not only the trapezohedral crystals from other tetragonal crystals, but we may distinguish right and left handed crystals. There is, therefore, a physical difference between prisms and pyramids, which are geometrically identical.

#### PYRAMIDAL HEMIHEDRAL CLASS

This type of hemihedrism corresponds in a measure to the pentagonal hemihedrism of the cubic system. One-half of the faces within an octant are selected but when passing from an upper to a lower octant the alternation is not continued. The alternate selection is strictly adhered to except for this departure. The holohedra which give rise to new forms by this method of selection must contain an even number of faces within the octant and the faces may not

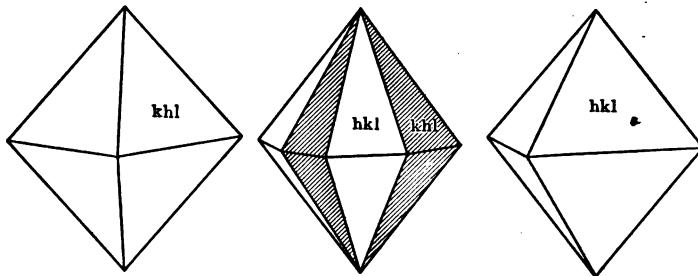


Fig. 80

extend from one upper octant into another upper octant or from one lower octant into another lower octant. A face may, however, extend from an upper into a lower. With these restrictions, it is readily apparent that the only two forms which can be productive of geometrically new types are the ditetragonal pyramid and the ditetragonal prism. Figure 80 indicates this method of selection as applied to the ditetragonal pyramid. It will be observed that four faces survive from the upper octants and that the four faces immediately beneath

represent the lower octants. Two congruent forms result. Geometrically they resemble the pyramids of the first and

second orders and are called pyramids of the third order. These three types of pyramid differ from one another chiefly in orientation as shown in Figure 81, which represents diagrammatically a cross-section of these three pyramidal types. If one of the pair of congruent forms shown in Figure 80 be turned about its tetrad axis through a certain angle it assumes an

orientation identical with the other form. They are distinguished from one another as right and left and possess the following symbols:

1. Left —  $\pi(hkl)$
2. Right —  $\pi(khl)$

This method of selection applied to the ditetragonal prism as indicated in Figure 82 results in the suppression of one-

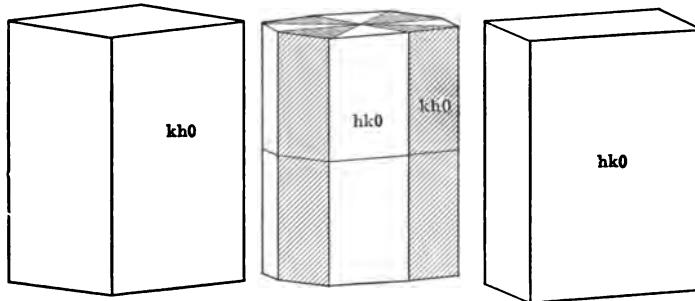


Fig. 82

half the prismatic faces and the extension of the other half. Two new forms — prisms of the third order — are obtained, geometrically identical with the prisms of the first and second order which differ from them only in orientation. They are:

1. Left —  $\pi(hk0)$
2. Right —  $\pi(kh0)$

The symmetry of forms belonging to this class is as follows:

One chief plane,  
One tetrad axis,  
Centre present.

The pyramids and prisms of the third order occur in combination with certain holohedra of this system. In this as in the last class the physical properties of the crystals are distinct from those of the holohedral class, even though they should be bounded only by apparently holohedral forms. The minerals scheelite,  $\text{CaWO}_4$ , scapolite (Figure 83), fergus-

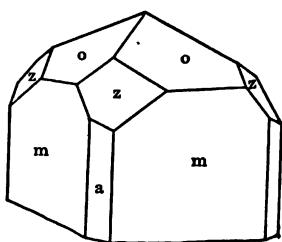


Fig. 83—Scapolite showing the prism of the first order,  $m$ , nite showing the prism of the second of the third order,  $a$ , (100); the pyramid  $\pi$  (320); the pyramid  $\pi$  (321); the pyramid of the first order,  $o$ , (111) and of the third order,  $z$ , of the third order,  $g$ , of the first order,  $s$ , (111) and the basal pinacoid,  $c$ , (001).

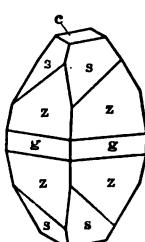


Fig. 84—Fergusonite showing the prism of the second of the third order,  $s$ , (111) and the basal pinacoid,  $c$ , (001).

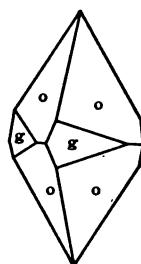


Fig. 85—Stolzite showing the prism of the first order,  $o$ , (111) and the basal pinacoid,  $c$ , (001).

sonite (Figure 84) and stolzite,  $\text{PbWO}_4$  (Figure 85) crystallize in this class.

#### SPHENOIDAL HEMIHEDRAL CLASS

A third hemihedral class results from the selection of one-half the faces by octants. Half the octants are selected alternately and all the faces contained within the selected octants are extended. Those contained in the other four octants are suppressed. Geometrically new forms may be obtained here only from those holohedra where faces are confined to a single octant. Where a face is common to two octants, new forms will not result by the application of this method. Only two holohedra, therefore, are capable of giv-

ing rise to hemihedral forms of this class, viz., the pyramid of the first order and the ditetragonal pyramid. Figure 86 indicates the application of this method of selection to the ditetragonal pyramid.

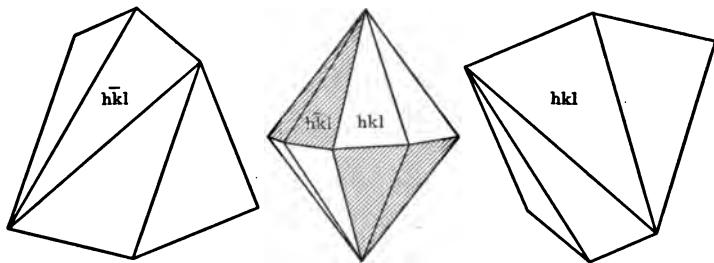


Fig. 86

cates the application of this method of selection to the ditetragonal pyramid.

From the pyramid of the first order are obtained two congruent forms each enclosed by four isosceles triangles. These forms differ from one another only in orientation, while they differ from the tetrahedron in that their faces are isosceles instead of equilateral triangles. These hemihedral forms which are called tetragonal sphenoids of the first order are symbolized thus:

1. Positive —  $\kappa(hhl)$
2. Negative —  $\kappa(h\bar{h}l)$

Figure 87 represents a combination of the positive and negative forms.

From the ditetragonal pyramid a pair of congruent forms is obtained. Each form is bounded by eight faces and has a

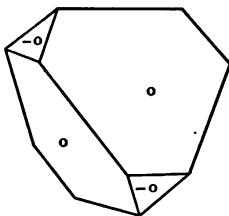


Fig. 87 — Chalcopyrite showing the positive sphenoid  $\kappa(111)$  and the negative sphenoid,  $-\kappa(111)$ .

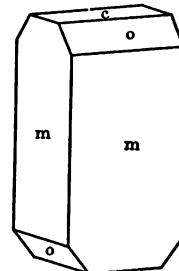


Fig. 88 — Urea showing the prism of the first order,  $m$ , (110); the basal pinacoid,  $c$ , (001) and the positive sphenoid,  $\kappa$ ,  $\kappa(111)$ .

general resemblance in its outline to the tetragonal sphenoid, a pair of faces here replacing a single face of the last type. They are known as tetragonal scalenohedra and are distinguished from one another as follows:

1. Positive —  $\kappa(hkl)$
2. Negative —  $\kappa(h\bar{k}l)$

The symmetry for forms of this class is:

Two intermediate planes,

Two diad axes corresponding in direction to the axes of reference  $a_1$  and  $a_2$ ,

One diad axis parallel to the vertical axis  $c$ .

This type of hemihedrism is exhibited on crystals of chalcopyrite,  $\text{CuFeS}_2$  (Figure 87), and of urea,  $\text{CH}_4\text{N}_2\text{O}$  (Figure 88).

#### TETARTOHEDRAL CLASS

The simultaneous application of two methods of hemihedrism gives rise to forms possessing one-quarter of the faces characteristic of certain holohedra. Figures 79, 80 and 86 indicate the method of selection characteristic of the three hemihedral processes. The simultaneous application of the sphenoidal and pyramidal methods of selection to the ditetragonal pyramid produces the bisphenoids of the third order. These forms resemble the tetragonal sphenoids already described. They are four in number according to orientation and possess the following symbols:

1. Positive right —  $\kappa\pi(hkl)$ ,
2. Positive left —  $\kappa\pi(h\bar{k}l)$ ,
3. Negative right —  $\kappa\pi(h\bar{k}l)$ ,
4. Negative left —  $\kappa\pi(k\bar{h}l)$ .

Similarly from the pyramid of the second order two sphenoids of the second order are obtained. This same method of selection, when applied to the ditetragonal prism and to the pyramid of the first order, produces a prism of the third order and the sphenoid of the first order identical in all respects with those already described. Forms belonging to this class are symmetrical about only one diad axis corresponding in direction with the vertical axis for tetragonal crystals. Up to the present time no substances, either

natural or artificial, have been recognized as belonging to this class.

#### HOLOHEDRAL HEMIMORPHIC CLASS

Certain substances crystallizing in the tetragonal system are peculiar in that the faces which intersect the vertical axis at the positive end do not correspond to those intersecting this axis at the negative end. They are said to be hemimorphic and the vertical axis is called a hemimorphic axis. There are two classes of such crystals belonging to this system. One, known as the holohedral hemimorphic class, presents the following symmetry:

*Axes.* One tetrad,

*Planes.* Two secondary and two intermediate,

*Centre absent.*

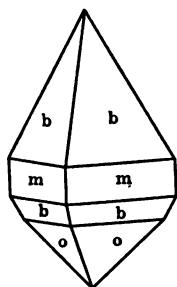


Fig. 89 — Iodosuccinimide showing the prism of the first order,  $m$ , (110); the upper and lower halves of the pyramid of the first order,  $b$ , (221) and  $(22\bar{1})$ ; the lower half of the pyramid of the first order,  $o$ ,  $(11\bar{1})$ .

This class represents a symmetry which would result by starting with the holohedral class and dropping the chief plane of symmetry, which would incidentally carry with it the centre and the four diad axes. The name given suggests its derivation in this fashion from the holohedral class.

The pyramids and pinacoid of the holohedral class fall into pairs of hemimorphic forms, one designated as upper and the other lower. These may be symbolized as follows:

- Pyramids:* 1. First order, upper (hh $l$ ) and lower (hh $\bar{l}$ )
- 2. Second order, upper (h0l) and lower (h0 $\bar{l}$ )
- 3. Ditetragonal, upper (hkl) and lower (h $k\bar{l}$ )

*Basal pinacoids.* The basal pinacoid, upper (001) and lower (00 $\bar{1}$ ).

Iodosuccinimide crystallizes in this class (Figure 89).

#### HEMIHEDRAL HEMIMORPHIC CLASS

A second hemimorphic class of the tetragonal system stands in the same relationship to the pyramidal hemihedral class

as the holohedral hemimorphic occupies with reference to the holohedral class. The various forms other than the prisms are designated as upper and lower which is shown in the symbols by the sign placed over the last index to indicate which end of the vertical axis is intersected by the form in question. The tetrad axis is the only element of symmetry characteristic of these forms. Figure 90 indicates the characteristic combination on a crystal of wulfenite,  $\text{PbMoO}_4$ .

Crystals of these two hemimorphic classes behave in a peculiar way when heated or cooled. It is found that with the change of temperature the faces grouped around opposite ends of the vertical axis become electrified. In a particular crystal that end which becomes positively electrified with the rise of temperature becomes negatively electrified with the fall of temperature. The reverse holds for the other end. Such crystals are said to be pyro-electric. This phenomenon, which may be observed in crystals of many classes, is most marked on those crystals which are characterized by the presence of an axis of symmetry which is at the same time a hemimorphic axis. In order to detect the presence of electrified surfaces resulting from change in temperature a mixture of red lead and sulphur may be employed. If this be shaken in a small bellows so that the particles become electrified by friction and blown into the atmosphere the particles positively electrified are attracted to faces on the crystal showing the negative sign, while the particles negatively electrified are attracted to the faces with the opposite sign. In this way some of the electrified surfaces on the crystal become red and others become yellow.

**Projections.** Figure 91 shows a gnomonic projection parallel to 001 for the mineral rutile. The points upon this projection are distributed symmetrically about the pole and also about four lines passing through the pole and at angles of  $45^\circ$  to one another. These four lines of symmetry exhibited

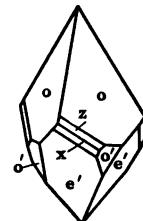


Fig. 90 — Wulfenite showing the upper and lower halves of the pyramid of the first order,  $o$ , (111) and  $o'$  ( $1\bar{1}\bar{1}$ ); the lower half of the pyramid of the second order,  $e'$ , ( $10\bar{1}$ ); the half pyramids of the third order,  $x$ ,  $\pi$  (311) and  $z$ ,  $\pi$  (432).

by the projection correspond to the trace of the two secondary and two intermediate planes of symmetry characteristic of the holohedral class. The height of the plane of projection above the centre of the crystal is marked by the ground circle whose radius corresponds to the height. It will be observed here that the fore and aft pace is identical with the

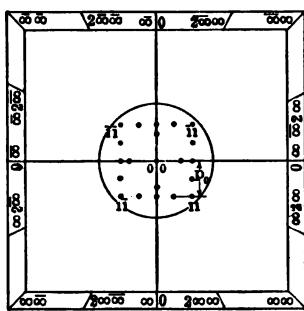


Fig. 91

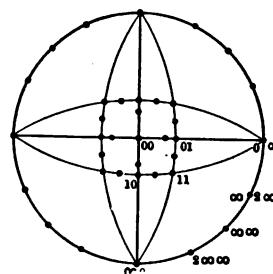


Fig. 92

right and left pace. This characteristic tetragonal pace is designated  $p_0$  which in the tetragonal system is irrationally related to  $r_0$  — the height of the plane of projection above the centre, here marked by the radius of the ground circle. Before it is possible to write the symbols for any of the faces exhibited on this projection it is necessary arbitrarily to assign symbols to one or more faces so as to fix the value of  $p_0$ . This having been done the symbols for all the faces present can readily be worked out. Figure 92 is a stereographic projection for rutile exhibiting the same faces as are shown on the gnomonic projection.

## CHAPTER X

### HEXAGONAL SYSTEM

In the hexagonal system are included twelve classes. Many of the commonest substances crystallize in this system and in some respects, both on account of the number of substances crystallizing in it, and on account of the varying degrees of symmetry exhibited, this must be regarded as the most important system.

**Symmetry.** Crystals of the holohedral class exhibit the following symmetry:

- (a) *Axes.* One hexad — three diad axes lying in a plane at right angles to the hexad axis. These three axes make with one another angles of  $60^\circ$  and  $120^\circ$ . In this same plane three axes of symmetry alternate with the three previously referred to. Three of these axes are secondary and three intermediate axes of symmetry.
- (b) *Planes.* One chief plane containing six axes of symmetry — this plane is at right angles to the hexad axis; three planes, each containing the hexad axis and one secondary axis of symmetry; three planes each containing the hexad axis and one intermediate axis of symmetry.
- (c) *Centre of symmetry present.*

These various elements of symmetry are diagrammatically illustrated in Figures 93 and 94.

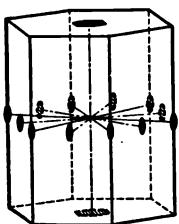


Fig. 93

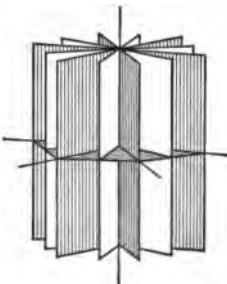


Fig. 94

**Standard Orientation.** Crystals belonging to the hexagonal system are by agreement oriented so that the hexad axis

is vertical while a diad axis is right and left. As there are two sets each containing three diad axes, there is for each substance crystallizing in the hexagonal system a choice as to the set from which the right and left axis shall be chosen. The crystallographer who first describes the crystals of a substance arbitrarily makes the choice and his orientation of the crystal is afterwards strictly observed. The three diad axes of symmetry, one of whose members occupies the right and left position, are known as secondary axes. The three diad axes of the other set are intermediate.

**Axes of Reference.** Faces of all hexagonal crystals, regardless of the class to which they belong, may be referred to three horizontal axes coincident in direction with the three secondary axes of symmetry and one vertical axis which coincides with the hexad axis of symmetry. The three horizontal axes of reference are equivalent, that is, any plane which intersects two or three of them meets the axes so as to form intercepts whose lengths are simply and rationally related to one another. Since these three axes are equivalent they are designated  $a_1$ ,  $a_2$  and  $a_3$  with the peculiar order and distribution of signs exhibited in

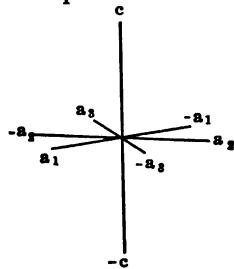


Fig. 95

Figure 95. The vertical axis designated by the letter  $c$  is unequal to the three lateral axes since any crystal face intersecting the vertical and one or more lateral axes cuts off on the vertical axis a length which is irrationally related to the length on any one of the lateral axes. This equivalence of three axes of reference lying in the horizontal plane and their inequivalence to the vertical axis

$c$  corresponds closely to the relationship observed in the tetragonal system.

**Classification of All Possible Planes Intersecting Hexagonal Axes of Reference.** Certain planes have such directions as to intersect the vertical axis at finite distances while others intersect the vertical axis only at infinity, because there lies in these planes a line which is parallel to the vertical axis. These two main subdivisions may be still further separated as follows:

- (a) The plane intersects vertical axis:
  - 1. Lateral axes not cut,
  - 2. Three lateral axes cut unequally,
  - 3. Three lateral axes cut, two equally, one at half the distance,
  - 4. Two lateral axes cut equally, the third at infinity.
- (b) The plane does not intersect the vertical axis:
  - 1. Three lateral axes cut unequally,
  - 2. Three lateral axes cut, two equally, the third at half the distance,
  - 3. Two lateral axes cut equally, the third at infinity.

This classification presents to us in the hexagonal system seven possibilities, each of which corresponds to a type of crystal form similar to the types studied in connection with the previous two systems. The following table gives the names of the forms in the hexagonal system and indicates the nature of their intersection of the axes of reference:

- 1. Vertical axis cut, two lateral axes cut equally, the other at infinity — pyramid of the first order,
- 2. Vertical axis cut, two lateral axes cut equally, the other at half the distance — pyramid of the second order,
- 3. Vertical axis cut, three lateral axes cut unequally — dihexagonal pyramid,
- 4. Vertical axis cut, lateral axes not cut — basal pinacoid,
- 5. Vertical axis not cut, two lateral axes cut equally the other at infinity — prism of the first order,
- 6. Vertical axis not cut, two lateral axes cut equally, the other at half the distance — prism of the second order,
- 7. Vertical axis not cut, lateral axes cut unequally — dihexagonal prism.

**Hexagonal Symbols.** In this system there are three equivalent axes of reference lying in one plane and a fourth axis of a different kind which is normal to the plane of these three. Hence it will be necessary in writing general symbols to introduce a fourth letter. For all other systems only three letters are required. The fourth letter required for general symbols of the hexagonal system is *i* and stands in the following relationship:  $h > k > i > l$ . It will be particularly noted that this is not the alphabetical order. It would be

quite possible to indicate the direction of hexagonal planes by using only two indices for the lateral axes since when the indices of two of the lateral axes is known the third one may be calculated. The general relationship existing between the lengths of the intercepts of the three lateral axes may be expressed as follows: if the shortest intercept be considered unity, the longest intercept  $n$ , then the intercept of the third axis is  $\frac{n}{n-1}$ . This may be established by reference to Figure 96 as follows:

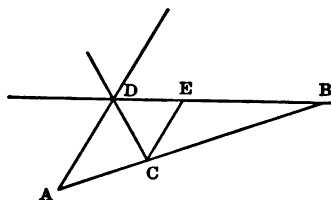


Fig. 96

Let AB be a plane intersecting the three lateral hexagonal axes of reference in the points A, B and C, giving the three intercepts AD, BD and CD. The shortest of these  $CD = 1$ ; the longest  $BD = n$ . From C draw CE parallel to AD. Then the triangles ABD and CBE are similar

$$\text{similar and } AD: CE :: BD: BE \therefore AD = \frac{CE \cdot BD}{BE} = \frac{1 \cdot n}{n-1} = \frac{n}{n-1}$$

Since the plus and minus signs are arranged alternately on the axes of reference of the hexagonal system it follows that a plane which intersects the lateral axes necessarily intersects both positive and negative ends of these axes. Keeping in mind this alternation of positive and negative signs and of the parameter ratio  $\frac{n}{n-1} a_1: na_2: -1a_3$  we con-

clude that the algebraical sum of the indices for the three lateral axes must in all hexagonal symbols be zero. This may be established by conversion of the above general parameter ratios into the index symbols of Miller:

$$\text{Parameter ratio, } \frac{n}{n-1} a_1: na_2: -1a_3,$$

$$\text{Reciprocal ratio, } \frac{n-1}{n} : \frac{1}{n} : -\frac{1}{1},$$

$$\text{Indices, } (n-1): 1: -n$$

In the hexagonal system, therefore, when the indices of two lateral axes are known the third may be readily derived

by assigning to it such a value that the sum of all three indices becomes zero. This theorem may be employed to check the correctness of hexagonal symbols; thus symbols such as the following are manifestly impossible: (4311), (2232).

#### HOLOHEDRAL CLASS

**Pyramid of the First Order.** The faces of this type intersect two lateral axes at equal distance from the centre and are parallel to the third. The intercept on the vertical axis, in accordance with the characteristics of this system, is irrationally related to the intercepts on the lateral axes. There are many possible forms of this pyramid, some steeper intersecting the vertical axis at a greater distance, others flatter with a shorter intercept on this axis. The one who first describes a crystal of a hexagonal substance arbitrarily selects one of these pyramids as the unit or ground pyramid. Its symbol, therefore, becomes (1011). The ratio between the lengths of the intercepts on the vertical and lateral axes for this pyramid gives us the ratio  $c:a$ , which becomes a characteristic constant for all crystals of this substance. The value for  $c$  may be either greater or less than that for  $a$ , thus, for beryl —

$$c:a :: .8643:1,$$

for corundum —

$$c:a :: 1.3636:1.$$

In different hexagonal substances the ratio  $c:a$  varies but it is always irrational. This corresponds to the condition which maintains in the tetragonal system. The hexagonal pyramid of the first order (Figure 97) is enclosed by twelve faces each of which in the idealized form, where all the faces occur at equal distance from the centre, is an isosceles triangle. There are twelve polar edges and six basal edges. The corners are of two kinds, two hexahedral and six tetrahedral. The symbol (h0 $\bar{h}$ l) is capable of representing all possible pyramids of the first order.

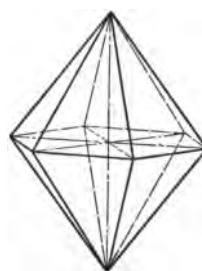


Fig. 97

**Pyramid of the Second Order.** The faces of this type intersect two lateral axes at equal distances from the centre,

the third at one-half the distance. The vertical axis may be intersected at varying distances, in this way giving rise to many individual forms, some flatter, some steeper. Geometrically the pyramid of the second order is similar to the one

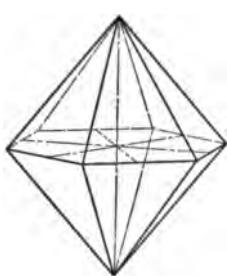


Fig. 98

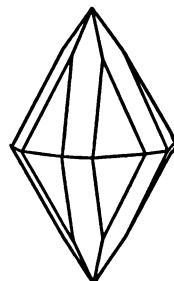


Fig. 99

already described, but differs from it in orientation (Figure 98). The relationship between these two pyramids is the same as that which holds between the pyramids of the first and second orders of the tetragonal system. The symbol for the unit pyramid of the second order is  $(11\bar{2}1)$ , while that for all pyramids of this type is  $(kk\bar{h}l)$ , in which  $h = 2k$ . These pyramids of the first and second order are represented in combination in Figure 99.

**Dihexagonal Pyramid.** Each face of this type cuts all three of the lateral axes at different distances from the centre

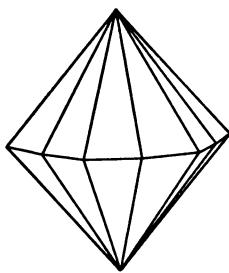


Fig. 100

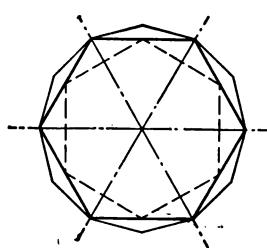


Fig. 101

and the vertical axis at a distance which is irrationally related to the lengths on the lateral axes. The symmetry of this class requires twenty-four faces of this kind to constitute the crystal form. Each face is a scalene triangle. There are

twelve basal edges and twenty-four polar edges — two sets of twelve each. Two corners representing the highest and lowest points of the crystal are dodecahedral, while there are twelve tetrahedral corners, two sets of six each marking the points of exit of the secondary and intermediate axes of symmetry. This is the most general form of the hexagonal system, with the general symbol (kih̄l) (Figure 100). The dihexagonal pyramids, more frequently observed on the mineral beryl, are the following: (21̄33), (51̄65), (54̄94), (21̄31), (31̄41), (51̄61) and (42̄61).

The mode of intersection on the lateral axes of the three pyramids is shown in Figure 101.

**Hexagonal Prisms.** In this system, as in the tetragonal system, there are three prisms corresponding in cross-section to the three pyramids. They are designated —

1. Prism of the first order with the symbol (10̄10),
2. Prism of the second order with the symbol (11̄20),
3. Dihexagonal prism with the symbol (kih̄0).

The first and second of these prisms are hexagonal in cross-section and differ from one another only in orientation. Each of these prisms is bounded by six planes meeting in six edges parallel to the vertical axis. The cross-section of the dihexagonal prism is twelve-sided. A dihexagonal prism in which all twelve angles are equal would involve irrationality of the axial parameters, and consequently is not crystallographically possible. This form is bounded by twelve planes meeting in twelve edges parallel to the vertical axis. All these prisms are open forms and consequently occur on crystals only in combination with one or more of the pyramids or with the basal pinacoid. Figures 102, 103 and 104 represent the three prisms in combination with the next form.

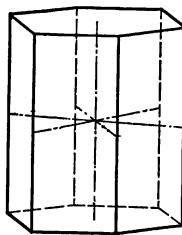


Fig. 102

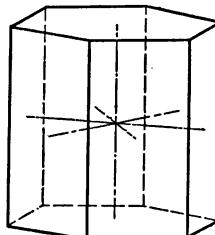


Fig. 103

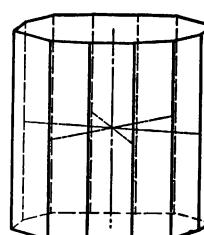


Fig. 104

**Basal Pinacoid.** This form consists of two faces parallel to one another occurring at opposite ends of the vertical axis. Symbol (0001). This is an open form which may occur on crystals in combination with prisms and pyramids.

**Limiting Forms.** The limitation of the various forms in the hexagonal system is in most respects closely related to that already observed for the tetragonal system. Certain pyramids are steeper than the unit pyramid, others flatter. The former may be supposed to find their limit in the prism, the infinitely steep pyramid, and the latter in the pinacoid, the infinitely flat pyramid. With regard to the intercepts on the lateral axes variations may reach their limits in two ways: first  $k$  may increase in value until it is equal to  $h$  when the pyramid or prism of the first order results; second,  $k$  may diminish in value until it is equal to one-half  $h$ , so giving rise to the pyramid or prism of the second order. The relationships of the pyramids to the prisms and basal pinacoid and of the dihexagonal forms to the forms of the first and second order may be diagrammatically expressed in

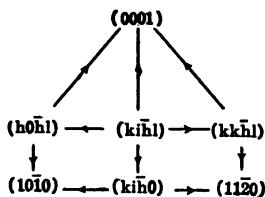


Fig. 105

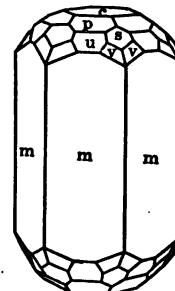


Fig. 106 — Beryl showing the prism of the first order,  $m$ , (1010); the basal pinacoid,  $c$ , (0001); the pyramids of the first order,  $u$ , (2021) and  $p$ , (1011); the pyramid of the second order,  $s$ , (1121) and the dihexagonal pyramid  $v$ , (2131).

Figure 105. Beryl (Figure 106), pyrrhotite and langbanite crystallize in the holohedral hexagonal class.

**Calculation of the Ratio  $c:a$ .** This can be most simply calculated in the hexagonal system from the basal angle between two faces of the unit pyramid of the second order

(1122). When this is known the value of  $c$  ( $a$  being unity) is the tangent of half the inner angle. From various other angular measurements this ratio may be determined although not so simply. The relationship between the pace distance  $p_0$  and  $c$  is expressed as follows:

$$p_0 = \frac{2}{3} c.$$

**Projections.** Figure 107 represents a gnomonic projection of beryl as shown in Figure 106. An examination of this projection indicates that the points are symmetrically distributed with regard to six lines of symmetry and that these lines pass through the pole which is a point of hexad symmetry. The pace lengths measured in the direction of the three lines of symmetry, which correspond to the axes of reference

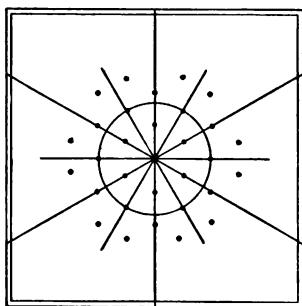


Fig. 107

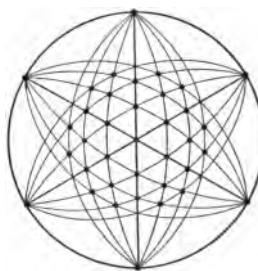


Fig. 108

$a_1, a_2, a_3$ , are equivalent for all three directions and are consequently designated  $p_0$ . The length  $p_0$  is irrationally related to  $r_0$ , whose magnitude is represented by the radius of the ground circle. Figure 108 is a stereographic projection of the forms shown in the gnomonic projection (Figure 107).

#### HOLOHEDRAL HEMIMORPHIC CLASS

This class may be easily derived from the holohedral class by dropping the chief plane of symmetry. This means the simultaneous disappearance of the centre and of the six diad axes. Although the hexad axis remains, the forms which meet one end of this axis do not necessarily intersect the opposite end in a similar fashion. The symmetry of this

class is therefore one hemimorphic hexad axis, three secondary planes of symmetry and three intermediate planes. On crystals of this class the three prisms characteristic of holohedrism occur but may be distinguished from the corresponding holohedral forms by means of etching figures. The other four forms, viz., basal pinacoid, dihexagonal pyramid and the pyramids of the first and second orders are each divided

into two forms designated respectively upper and lower, with the following symbols:

1. Pyramid of the first order, upper ( $h0\bar{h}l$ ),
2. Pyramid of the first order, lower ( $h0\bar{h}\bar{l}$ ),
3. Pyramid of the second order, upper ( $kk\bar{h}l$ ),
4. Pyramid of the second order, lower ( $kk\bar{h}\bar{l}$ ),
5. Dihexagonal pyramid, upper ( $ki\bar{h}l$ ),
6. Dihexagonal pyramid, lower ( $kih\bar{l}$ ),
7. Basal pinacoid, upper ( $0001$ ),
8. Basal pinacoid, lower ( $000\bar{1}$ ).

Fig. 109—Zincite showing the prism of the first order,  $m$ , ( $10\bar{1}0$ ); the upper half pyramid of the first order,  $p$ , ( $10\bar{1}1$ ) and the lower basal pinacoid, ( $000\bar{1}$ )

The mineral zincite which belongs to this class is illustrated in Figure 109.

#### RHOMBOHEDRAL HEMIHEDRAL CLASS

Rhombohedral hemihedrism results from the extension of the faces contained in alternate dodecants with the suppression of those faces occurring in the other dodecants. The

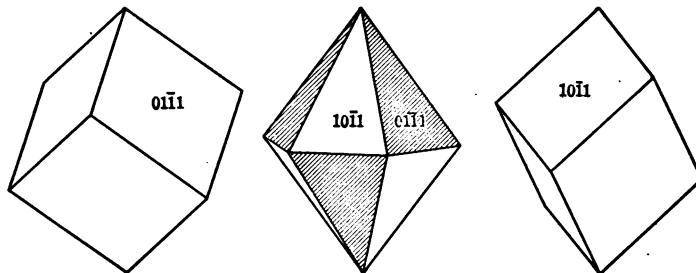


Fig. 110

only forms of the hexagonal system which produce geometrically new forms by this method of selection are those whose faces are confined within a single dodecant — the pyramid of the first order and the dihexagonal pyramid. Crystals of

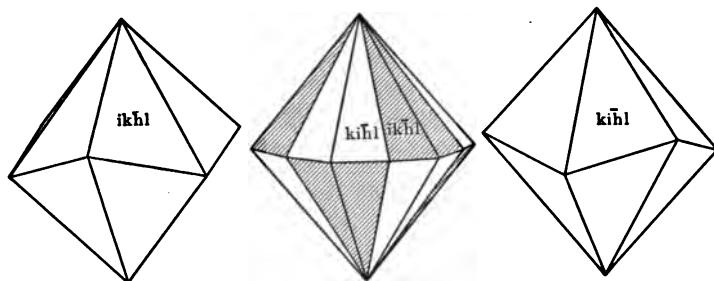


Fig. 111

this class are characterized by the presence of one triad axis and three diad axes, three intermediate planes and a centre of symmetry. This method of hemihedral selection is shown in Figures 110 and 111.

**Rhombohedron.** The application of this method of selection to the pyramid of the first order (Figure 110) gives rise to a pair of rhombohedra — congruent forms — each of which is bounded by six faces. The faces are all rhombs in outline. There are six polar edges and six basal edges. In certain

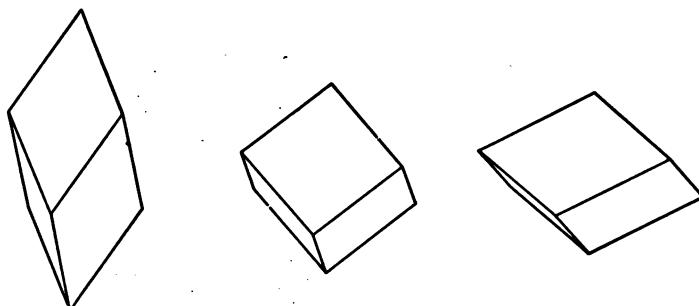


Fig. 112 — The cube as the intermediate form between flat and steep rhombohedra.

rhombohedra the angle over the polar is greater than that over the basal edge; in others the opposite relationship holds. In the flat rhombohedra the longer diagonal of the

rhomb is horizontal; in the steep rhombohedra this position is occupied by the shorter diagonal. This type of hemihedrism being indicated by the letter  $\kappa$ , the symbols for these two types of rhombohedra are:

1. Positive rhombohedron — symbol of the unit form  $\kappa(10\bar{1}1)$ , general symbol  $\kappa(h0\bar{h}l)$ ,
2. Negative rhombohedron — symbol of the unit form  $\kappa(01\bar{1}1)$ , general symbol  $\kappa(0h\bar{h}l)$ .

A rhombohedron whose basal and polar angles are equal to each other would be geometrically identical with the cube. This may be regarded as the limiting form between flat and steep rhombohedra (Figure 112).

**Scalenohedron.** The application of this method of selection to the dihexagonal pyramid (Figure 111) gives rise to two congruent forms bounded by twelve faces, each face in outline a scalene triangle. There is a great variety of scalenohedra, certain of them being very flat while others are steep and sharp. Not only is there a variation in the intercept on the vertical axis but there is also variation in the ratios of intercepts on the lateral axes. The following are the symbols for the various scalenohedra:

Positive scalenohedron,  $\kappa(kih\bar{l})$ ,  
Negative scalenohedron,  $\kappa(ik\bar{h}l)$ .

The six planes, whose directions are indicated by the adjacent pairs of basal edges of the scalenohedron, if extended

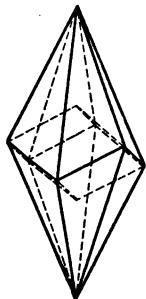


Fig. 113

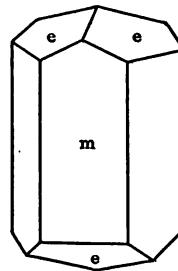


Fig. 114 — Calcite showing the prism of the first order,  $m$ ,  $(10\bar{1}0)$  and the negative rhombohedron,  $e$ ,  $\kappa(01\bar{1}2)$ .

to enclose space give rise to a rhombohedron, known as the rhombohedron of the middle edges. All those scalenohedra,

which intersect the lateral axes in the same fashion but differ from one another in varying lengths on the vertical axis, have basal edges identical in direction and are sometimes referred to as derivatives of the same rhombohedron of the middle edges. (Figure 113.)

A very large number of minerals crystallize in this class, some of the commonest of which are as follows: calcite,

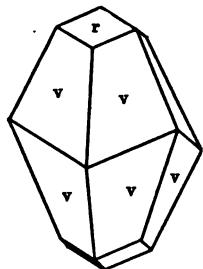


Fig. 115—Calcite showing the positive rhombohedron,  $r$ ,  $\kappa(10\bar{1}1)$  and the positive scalenohedron,  $v$ ,  $\kappa(21\bar{3}1)$ .

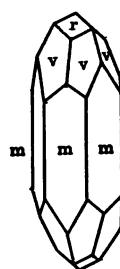


Fig. 116—Calcite showing the prism of the first order,  $m$ ,  $(10\bar{1}0)$ ; the positive rhombohedron,  $r$ ,  $\kappa(10\bar{1}1)$  and the scalenohedron,  $v$ ,  $\kappa(21\bar{3}1)$ .

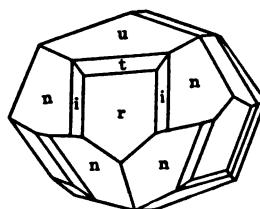


Fig. 117—Hematite showing the positive rhombohedra,  $t$ ,  $\kappa(30\bar{3}5)$ ,  $r$ ,  $\kappa(10\bar{1}1)$  and  $u$ ,  $\kappa(10\bar{1}4)$ ; the positive scalenohedron,  $i$ ,  $\kappa(42\bar{6}5)$  and the pyramid of the second order  $n$ ,  $(22\bar{4}3)$ .

$\text{CaCO}_3$  (Figures 114, 115 and 116), siderite,  $\text{FeCO}_3$ , corundum,  $\text{Al}_2\text{O}_3$ , and hematite,  $\text{Fe}_2\text{O}_3$  (Figure 117).

#### TRAPEZOHEDRAL HEMIHEDRAL CLASS

The selection and extension of one-half the faces within the dodecant and the regular alternation of the selection in adjacent dodecants gives rise to trapezohedral forms. Only the dihexagonal pyramid is capable of yielding geometrically new forms by this method of selection. Figure 118 shows the method of selection and the nature of the new forms resulting from it. These hexagonal trapezohedra are enantiomorphous and bear the general symbols:

Right hexagonal trapezohedron,  $\tau(kih\bar{l})$ ,

Left hexagonal trapezohedron,  $\tau(h\bar{i}kl)$ .

Crystals belonging to this class are devoid of centre and

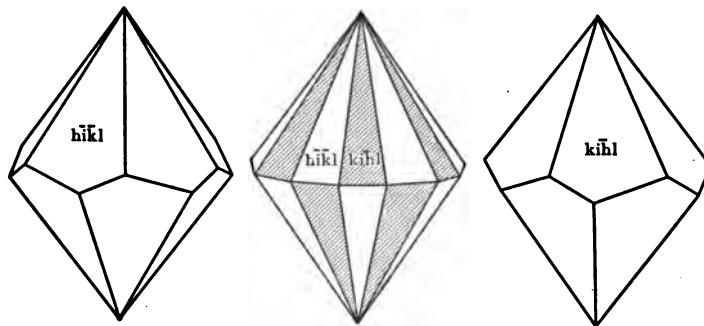


Fig. 118

planes of symmetry but exhibit the axes of symmetry which characterize the holohedral class.

#### PYRAMIDAL HEMIHEDRAL CLASS

The forms peculiar to this class of hemihedrism may be obtained by selecting and extending faces of the holohedra according to the method already indicated for the tetragonal pyramidal class. Only the dihexagonal pyramid and prism produce new forms according to this method of selection. Forms of this class are symmetrical about the chief plane, the hexad axis and the centre.

**Pyramid of the Third Order.** From the dihexagonal pyramid by this method of selection a congruent pair of pyramids of

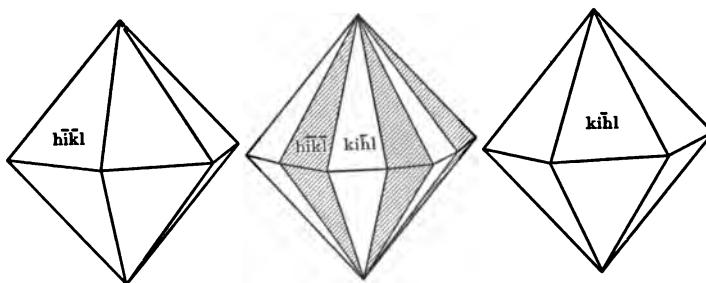


Fig. 119

the third order is obtained (Figure 119). These pyramids are geometrically identical with the pyramids of the first and second orders but differ from them in orientation. Symbols:

Positive pyramid of the third order,  $\pi(ki\bar{h}l)$ ,  
 Negative pyramid of the third order,  $\pi(h\bar{i}k\bar{l})$ .

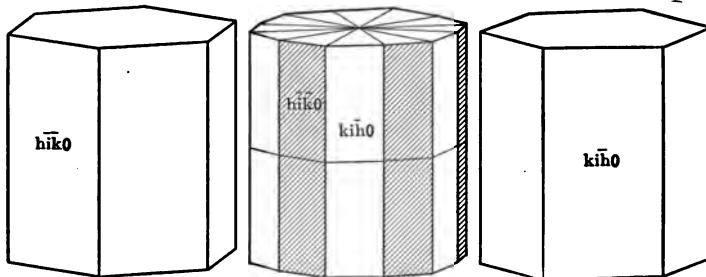


Fig. 121

The relationship of the pyramids of the three orders to one another is shown in Figure 120 by the way in which their faces intersect the axes of reference. The heavy lines represent the pyramid of the third order.

**Prism of the Third Order.** From the dihexagonal prism are obtained the positive and negative prisms of the third order. Their relationship to one another and to the prism from which they are derived is indicated in Figure 121. Symbols:

Positive prism of the third order,  $\pi(ki\bar{h}0)$ ,  
 Negative prism of the third order,  $\pi(h\bar{i}k0)$ .



Fig. 120

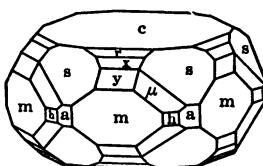


Fig. 122 — Apatite showing the three prisms—first order,  $m$ ,  $(10\bar{1}0)$ ; second order,  $a$ ,  $(11\bar{2}0)$  and third order,  $h$ ,  $\pi(21\bar{3}0)$ ; three pyramids of the first order,  $y$ ,  $(20\bar{2}1)$ ,  $x$ ,  $(10\bar{1}1)$  and  $r$ ,  $(10\bar{1}2)$ ; the pyramid of the second order,  $s$ ,  $(11\bar{2}1)$ , the pyramid of the third order,  $\mu$ ,  $\pi(21\bar{3}1)$  and the basal pinacoid,  $c$ ,  $(0001)$ .

The minerals of the apatite group exhibit this class of symmetry. Figure 122 represents combinations of the common form of apatite,  $3\text{Ca}_3(\text{PO}_4)_2 \cdot \text{Ca}(\text{F},\text{Cl})_2$ .

## PYRAMIDAL HEMIHEDRAL HEMIMORPHIC CLASS

This class whose only symmetry is that about a hemimorphic hexad axis may be regarded as a hemimorphic derivative of the last class. The crystal forms are the prisms of the first, second and third orders which are geometrically identical with those of the pyramidal hemihedral class and the basal pinacoid and pyramids of the first, second and third orders, but in this class these four forms are broken each into two forms designated upper and lower. The symbols for these eight forms are as follows:

Upper pyramid of the first order —  $(h0\bar{h}l)$ ,

Lower pyramid of the first order —  $(h0\bar{h}\bar{l})$ ,

Upper pyramid of the second order —  $(kk\bar{h}l)$ ,

Lower pyramid of the second order —  $(kk\bar{h}\bar{l})$ ,

Positive upper pyramid of the third order —  $(ki\bar{h}l)$ ,

Positive lower pyramid of the third order —  $(ki\bar{h}\bar{l})$ ,

Negative upper pyramid of the third order —  $(h\bar{i}kl)$ ,

Negative lower pyramid of the third order —  $(h\bar{i}k\bar{l})$ ,

Upper basal pinacoid —  $(0001)$ ,

Lower basal pinacoid —  $(000\bar{1})$ .

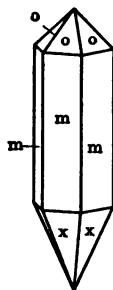


Fig. 123 — Strontium antimony dextro-tartrate showing the prism of the first order,  $m$ ,  $(10\bar{1}0)$ ; the upper hemipyramid of the first order,  $o$ ,  $(1011)$  and the lower hemipyramid of the first order,  $x$ ,  $(20\bar{2}1)$ . The corresponding lead compound gives rise to crystals of the same symmetry (Figure 123).

The mineral nepheline has been placed in this class as a result of examination of etching figures formed on certain faces. Strontium antimony dextro-tartrate,  $Sr_2(SbO)_2 \cdot (C_4H_4O_6)_2$  belongs, according to the goniometric measurements by Traube, to this type of symmetry. The

corresponding lead compound gives rise to crystals of the same symmetry (Figure 123).

## TRAPEZOHEDRAL TETARTOHEDRAL CLASS

Crystals of this class are characterized by the presence of one triad axis of symmetry and three hemimorphic diad axes. The characteristic tetartohedral forms may be derived by the simultaneous application of the methods of selection

by which the rhombohedral and trapezohedral classes of hemihedrism were obtained. This selection as applied to the

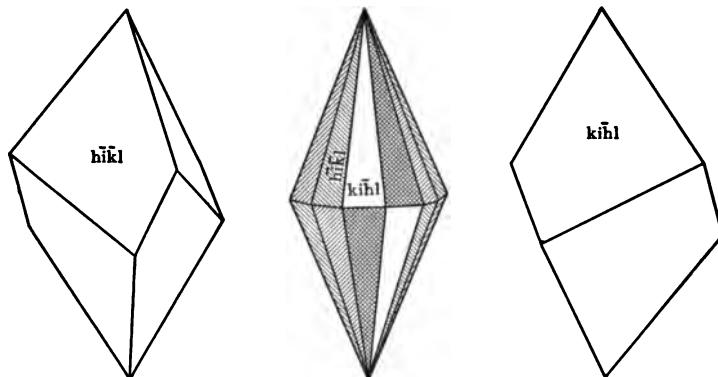


Fig. 124

dihexagonal pyramid is shown in Figure 124. The forms peculiar to this class are briefly referred to below:

**Trigonal Trapezohedron.** From the dihexagonal pyramid are obtained the two geometrically new forms illustrated in Figure 124. These forms show the right and left handed characteristics already observed in the hexagonal trapezohedra. They are enantiomorphous. In addition to these two other forms are obtained, one identical with each of those illustrated in Figure 124, but differing from it only in orientation. These four crystallographic forms therefore constitute two pairs of congruent forms. They are designated as follows:

1. Positive right trigonal trapezohedron  $\kappa\tau(kihl)$ ,
2. Negative right trigonal trapezohedron  $\kappa\tau(hkil)$ ,
3. Positive left trigonal trapezohedron  $\kappa\tau(hikl)$ ,
4. Negative left trigonal trapezohedron  $\kappa\tau(khil)$ .

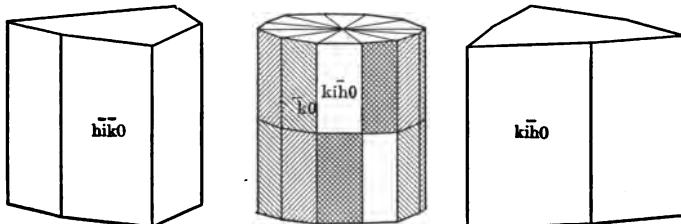


Fig. 125

**Ditrigonal Prism.** If this method of selection be applied to the dihexagonal prism two new forms are obtained, differing from one another only in orientation (Figure 125). These are designated:

1. Right ditrigonal prism —  $\kappa\tau(ki\bar{h}0)$ ,
2. Left ditrigonal prism —  $\kappa\tau(hi\bar{k}0)$ .

**Trigonal Pyramid.** From the pyramid of the second order a pair of congruent forms is obtained. They are known as:

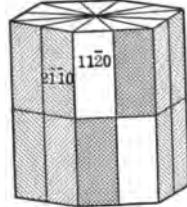


Fig. 126

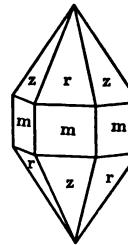


Fig. 127—Quartz showing the prism of the first order,  $m$ ,  $(10\bar{1}0)$  and the positive and negative rhombohedra of the first order simulating the pyramid of the first order,  $r$ ,  $\kappa(10\bar{1}1)$  and  $z$ ,  $\kappa(01\bar{1}1)$ .

1. Right trigonal pyramid, second order —  $\kappa\tau(kk\bar{h}l)$ ,
2. Left trigonal pyramid, second order —  $\kappa\tau(hk\bar{kl})$ .

**Trigonal Prism.** The same method of selection applied to the prism of the second order produces a pair of congruent

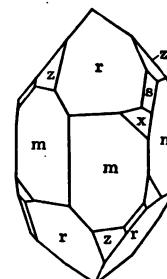
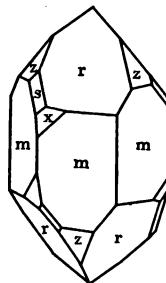


Fig. 128 and 129 — Quartz: the former left and the latter right handed. The forms common to the two crystals are the prism of the first order,  $m$   $(10\bar{1}0)$ ; the positive rhombohedron,  $r$ ,  $\kappa(10\bar{1}1)$ ; the negative rhombohedron,  $z$ ,  $\kappa(01\bar{1}1)$ ; and the trigonal pyramid,  $s$ ,  $\kappa\tau(2111)$ . On Figure 128 the left positive trigonal trapezohedron,  $x$ ,  $\kappa\tau(61\bar{5}1)$ ; on Figure 129 the right positive trigonal trapezohedron,  $x$ ,  $\kappa\tau(51\bar{6}1)$ .

forms known as trigonal prisms. Their relation to one another and to the prism of the second order from which they are obtained is shown in Figure 126. Symbols:

Right trigonal prism —  $\kappa\tau(11\bar{2}0)$ ,

Left trigonal prism —  $\kappa\tau(21\bar{1}0)$ .

The mineral quartz,  $\text{SiO}_2$ , shows the symmetry of this class although geometrically the faces present are usually so arranged as to suggest a higher symmetry (Figure 127). The occurrence, however, of the trigonal pyramids and trapezohedra is frequent, and from their position upon the crystals it is easy to determine whether the crystal is right or left. The faces are usually small and lie almost in the plane of the prism of the first right or left. The faces are usually either to the right or left, as illustrated in Figures 128 and 129, the former of which is left-handed and the other right-handed.

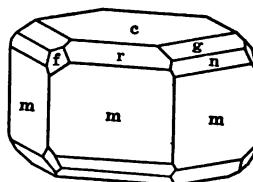


Fig. 130.—Cinnabar showing the prism of the first order,  $m$ ,  $(10\bar{1}0)$ ; the basal pinacoid,  $c$ ,  $(0001)$ ; the positive rhombohedron,  $r$ ,  $\kappa(10\bar{1}1)$ ; the negative rhombohedra,  $n$ ,  $\kappa(02\bar{2}1)$  and  $g$ ,  $\kappa(01\bar{1}2)$ ; the left positive trigonal trapezohedron,  $f$ ,  $\kappa\tau(83\bar{5}5)$ .

Cinnabar,  $\text{HgS}$  also crystallizes in this class. (Figure 130.)

#### RHOMBOHEDRAL TETARTOHEDRAL CLASS

In this class the crystals possess a triad axis and a centre of symmetry. The characteristic forms are such as might be obtained by the simultaneous application of the rhombohedral and pyramidal methods of hemihedral selection to certain holohedra of the hexagonal system. The only holohedra which give rise to new forms are the three pyramids and the dihexagonal prism.

**Rhombohedron of the First Order.** The application of this method of selection to the pyramid of the first order produces the rhombohedra of the first order (Figure 110). While geometrically identical with the rhombohedra previously described they differ from them physically.

**Rhombohedron of the Second Order.** By applying this method of tetartohedral selection to the pyramid of the second order a pair of rhombohedra is obtained geometri-

cally similar to those just described, but differing from them in orientation. Figure 131 shows the relation of these two forms to the pyramid of the second order. Symbols:

1. Positive rhombohedron of the second order  $\kappa\pi(k\bar{k}\bar{h}l)$ ,
2. Negative rhombohedron of the second order  $\kappa\pi(h\bar{k}kl)$ .

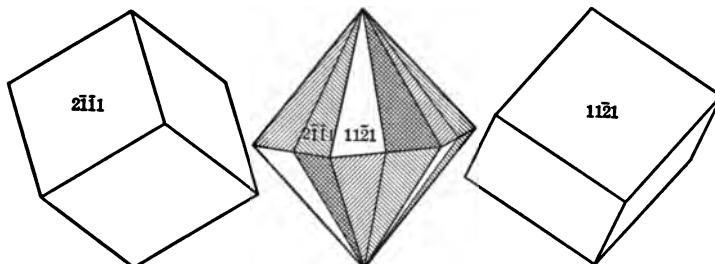


Fig. 131

**Rhombohedron of the Third Order.** Two pairs of rhombohedra geometrically similar to those of the first and second orders are obtained by this method from the dihexagonal pyramid. The derivation of two of these forms from the holohedral form is shown in Figure 132. The symbols for the four rhombohedra of the third order are as follows:

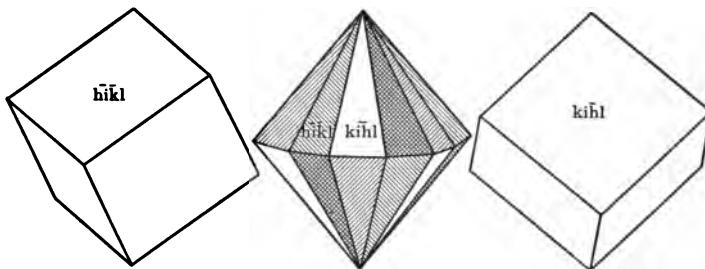


Fig. 132

1. Positive right rhombohedron of the third order —  $\kappa\pi(ki\bar{h}l)$ ,
2. Positive left rhombohedron of the third order —  $\kappa\pi(hikl)$ ,
3. Negative right rhombohedron of the third order —  $\kappa\pi(ihkl)$ ,
4. Negative left rhombohedron of the third order —  $\kappa\pi(ik\bar{h}l)$ .

**Prism of the Third Order.** From the dihexagonal prism two prisms of the third order are obtained. They are geometrically identical with the prisms obtained by pyramidal

hemihedrism. The minerals dioptase,  $\text{H}_2\text{CuSiO}_4$ , (Figure 133), phenacite (Figure 134), ilmenite and dolomite,  $(\text{Ca}, \text{Mg})\text{CO}_3$  (Figure 135) belong to this class.

The prisms of the first and second orders and the basal pinacoid do not change their form by the application of this

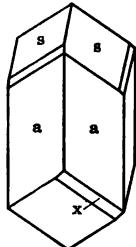


Fig. 133 — Diop-  
tase showing the prism of the second order,  $a$ ,  $(11\bar{2}0)$ ; the rhombohedron of the first order,  $s$ ,  $\kappa(02\bar{2}1)$  and the rhombohedron of the third order,  $x$ ,  $\kappa\pi(13\bar{4}1)$ .

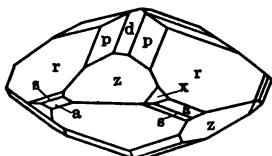


Fig. 134 — Phenacite showing the prism of the second order,  $a$ ,  $(11\bar{2}0)$ ; the positive second order,  $a$ ,  $(11\bar{2}0)$ ; the rhombohedra of the first order, the basal pinacoid,  $c$ ,  $z$ ,  $\kappa(10\bar{1}1)$  and  $d$ ,  $\kappa(10\bar{1}2)$ ; the  $(0001)$ ; the positive rhombohedron of the bohedron of the first order,  $r$ ,  $\kappa(01\bar{1}1)$ ; the order,  $r$ ,  $\kappa(10\bar{1}1)$ ; the positive pyramid of the second order,  $p$ ,  $(11\bar{2}3)$ ; the rhombohedron third order,  $y$ ,  $\kappa\pi(32\bar{5}1)$ ; of the third order,  $x$ ,  $\kappa\pi(21\bar{3}2)$ ; the rhombohedron of the third order,  $s$ ,  $\kappa\pi(21\bar{3}1)$ .

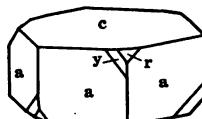


Fig. 135 — Dolomite showing the prism of the second order,  $a$ ,  $(11\bar{2}0)$ ; the basal pinacoid,  $c$ ,  $z$ ,  $\kappa(10\bar{1}1)$  and  $d$ ,  $\kappa(10\bar{1}2)$ ; the  $(0001)$ ; the positive rhombohedron of the bohedron of the first order,  $r$ ,  $\kappa(01\bar{1}1)$ ; the order,  $r$ ,  $\kappa(10\bar{1}1)$ ; the positive pyramid of the second order,  $p$ ,  $(11\bar{2}3)$ ; the rhombohedron third order,  $y$ ,  $\kappa\pi(32\bar{5}1)$ ; of the third order,  $x$ ,  $\kappa\pi(21\bar{3}2)$ ; the rhombohedron of the third order,  $s$ ,  $\kappa\pi(21\bar{3}1)$ .

method of selection. They occur on crystals of this class but are identical geometrically with those of the holohedral class.

#### TRIGONAL HEMIHEDRAL CLASS

The symmetry of this class consists of a triad axis, three hemimorphic diad axes, the chief plane and three intermediate planes of symmetry. The forms peculiar to this class may be obtained by selecting one-half the faces from the following holohedral forms: prism of the first order, dihexagonal prism, pyramid of the first order and dihexagonal pyramid.

In this method of selection the faces occurring in alternate dodecants above are extended while those in the three dodecants immediately below are extended at the same time. There are positive and negative forms.

**Trigonal Pyramid.** Two congruent forms are obtained from the pyramid of the first order by the selection indicated in

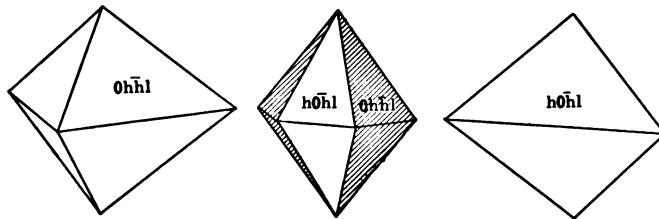


Fig. 136

Figure 136. They differ from the trigonal pyramids previously described principally in orientation.

1. Positive trigonal pyramid of the first order —  $(h0\bar{h}l)$ ,
2. Negative trigonal pyramid of the first order —  $(0h\bar{h}l)$ .

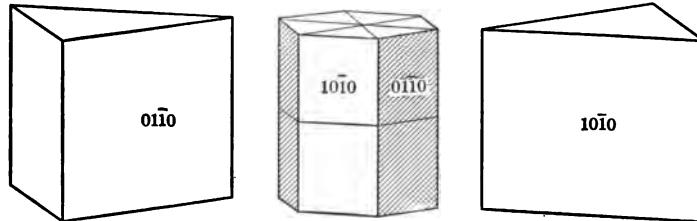


Fig. 137

**Trigonal Prism.** The selection by this method of one-half the faces of the prism of the first order (Figure 137) gives rise to two congruent forms:

Positive trigonal prism of the first order —  $(10\bar{1}0)$ ,  
 Negative trigonal prism of the first order —  $(01\bar{1}0)$ .

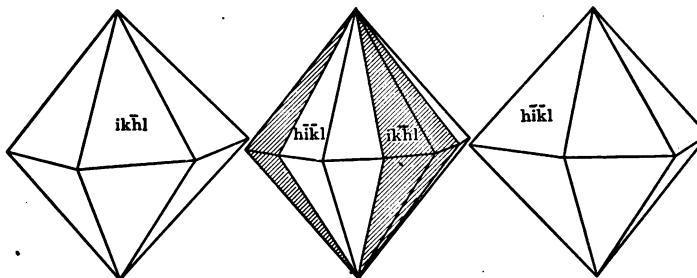


Fig. 138

**Ditrigonal Pyramid.** These forms are obtained from the dihexagonal pyramid. Their relations to one another and to the parent form are indicated in Figure 138.

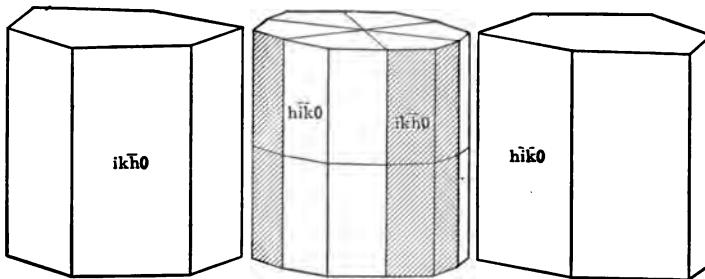


Fig. 138

Positive ditrigonal pyramid —  $(\bar{h}ik)$ ,

Negative ditrigonal pyramid —  $(ik\bar{h})$ .

**Ditrigonal Prism.** Figure 139 indicates the relationship of these two prisms to one another and to the dihexagonal prism.

Positive ditrigonal prism —  $(hik0)$ ,

Negative ditrigonal prism —  $(ik\bar{h}0)$ .

Most of the forms belonging to this class are geometrically similar to certain forms belonging to other classes, but differ from them in orientation.

Until recently no substance had been discovered crystallizing in this class, but in 1909 Rogers and Palache called attention to the symmetry of the new mineral benitoite as a first representative of this symmetry (Figure 140).

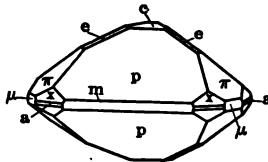


Fig. 140—Benitoite showing the trigonal prisms of the first order,  $m$  (1010) and  $\mu$ , (0110); the hexagonal prism of the second order,  $a$ , (1120); the trigonal pyramids of the first order,  $p$ , (1011);  $\pi$ , (0111);  $e$ , (0112); and the hexagonal pyramid of the second order,  $z$ , (2241). (After Palache.)

#### TRIGONAL HEMIHEDRAL HEMIMORPHIC CLASS

This class is symmetrical about one hemimorphic triad axis and about three intermediate planes. It may be conceived as a hemimorphic representative of the class just described. If from the last class the principal plane of sym-

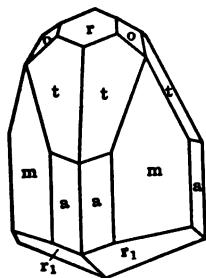


Fig. 141 — Tourmaline showing the trigonal prism of the first order,  $m$  (0110); the prism of the second order,  $a$ , (1120); the upper half forms of the following: the positive rhombohedron of the first order,  $r$ ,  $\kappa$  (1011); the positive scalenohedron,  $t$ ,  $\kappa$  (2131); the negative rhombohedron of the first order,  $o$ ,  $\kappa$  (0221); at the lower end the sole terminal form is the positive rhombohedron,  $\gamma$ ,  $\kappa$  (0111).

used for the demonstration of this phenomenon.

metry disappear and with it the three diad axes we obtain the degree of symmetry characteristic of this class. Its characteristic forms are therefore the same as those of the preceding class except that the basal pinacoid and the various pyramidal forms are here each represented by upper and lower forms.

The mineral tourmaline, a complex silicate, is the best known representative of this type of symmetry. Figure 141 exhibits the forms most frequently observed on crystals of this mineral. Its hemimorphism about the vertical axis is well marked. Like most hemimorphic minerals it becomes strongly pyro-electric with change of temperature and is commonly

#### TRIGONAL TETARTOHEDRAL CLASS

Crystals belonging to this class possess a chief plane and a triad axis of symmetry. The characteristic forms consist of trigonal pyramids and prisms of three different orders which may be derived by selecting one-quarter of the faces of the various pyramids and prisms possible in the holohedral

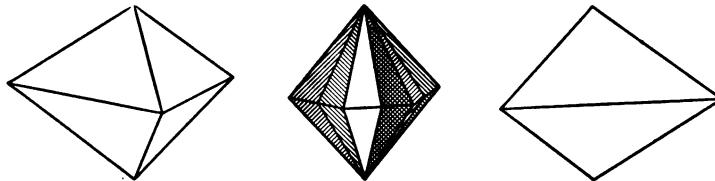


Fig. 142

hexagonal class. Those forms derived from the pyramid and prism of the first order are known as trigonal pyramids and

prisms of the first order. The pyramid and prism of the second order give rise to trigonal pyramids and prisms of the second order, while from the dihexagonal pyramids and prisms are obtained the corresponding trigonal pyramid and prism of the third order. The method of selection of one-quarter of the faces of the holohedra is shown in Figures 142 and 143. Since no substance has been discovered up to

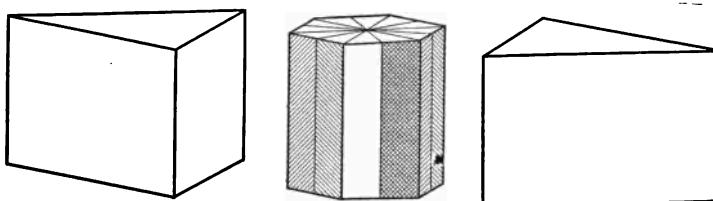


Fig. 143

the present time crystallizing in this class the development of the forms and their symbols is here omitted.

#### TRIGONAL TETARTOHEDRAL HEMIMORPHIC CLASS

Crystals of this class possess only a triad hemimorphic axis of symmetry coinciding with the vertical axis. This class may therefore be regarded as the hemimorphic representative of the preceding class. The basal pinacoid and the trigonal pyramids of the first, second and third orders are in this class each divided into a pair of forms — upper and lower. The trigonal prisms of the three orders may occur in the same geometrical form as in the last class, although the hemimorphism of the axis of symmetry might be determined by etching figures or by pyro-electricity. Sodium periodate,  $\text{NaIO}_4 + 3 \text{H}_2\text{O}$ , which crystallizes in this class is shown in Figure 144.

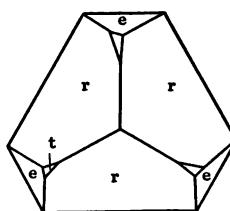


Fig. 144 — Sodium periodate — a plan showing the positive trigonal pyramid of the first order,  $r$ ,  $(10\bar{1}1)$ ; the negative trigonal pyramid of the first order,  $e$ ,  $(02\bar{2}1)$ ; the trigonal pyramid of the third order,  $t$ ,  $(21\bar{3}4)$ . The negative end of the vertical axis is cut by a large basal pinacoid  $(000\bar{1})$ .

## CHAPTER XI

### RHOMBIC SYSTEM

In the rhombic system are included three different classes each of which has a degree of symmetry peculiar to itself. Crystals belonging to these various classes are nevertheless closely related to one another in the following manner:

1. The forms characteristic of these classes can all be referred to three inequivalent axes at right angles to one another,
2. The forms characteristic of those classes of lower symmetry may be derived by regular selection of one-half the faces characteristic of some form belonging to the holohedral class,
3. There are certain holohedra belonging to the highest class which do not produce geometrically new forms by hemihedral or hemimorphic selection. These forms occur in combination with the characteristic forms in each of the classes belonging to the rhombic system. Such forms are the rhombic prism and pinacoids.

#### HOLOHEDRAL CLASS

**Symmetry.** Crystals of this class are symmetrical about three diad axes at right angles to one another, about three planes, each of which contains two axes of symmetry, and about a centre. The three axes, while possessing the same degree of symmetry are not equivalent, as each penetrates the crystal in a different fashion.

**Standard Orientation.** Rhombic crystals are so oriented that the three diad axes are respectively fore and aft, right and left and vertical. There are six possible ways of orienting a crystal so as to be in accord with this general restriction. The one who first describes a crystal of a substance

belonging to the rhombic system usually places it so that the diad axis corresponding to the greatest length of the crystal has the vertical position. In determining the direction of the other two axes, account is taken of the form of the cross-section of the crystal. In this system, as the name suggests, the rhombus is the common cross-section of crystals. When the vertical direction has been selected that axis of symmetry which corresponds to the shorter diagonal of the rhomb, in the form most prominently developed, is placed fore and aft, while the longer diagonal of the rhomb is right and left. It sometimes happens however that the crystals from which the substance was first described do not represent the ordinary development of the substance. Later it may be found that the axis which was selected for the vertical direction in reality usually corresponds to the shortest dimension of the crystal, or that rhomb which was most prominent in the cross-section and consequently decisive in indicating which of the other two axes should be right and left and fore and aft, does not as a rule correspond to faces of large development. Regardless of these later discoveries the orientation given to the crystal by the original investigator is retained, so that in certain rhombic crystals, as pictured in the text books, the vertical direction really corresponds to its shortest dimension. This is often observed on crystals of barite,  $\text{BaSO}_4$ , celestite,  $\text{SrSO}_4$ , and anglesite,  $\text{PbSO}_4$ .

**Axes of Reference.** The axial cross for crystals of the rhombic system consists of three lines at right angles to one another, coinciding with the directions of the three axes of symmetry. A crystal face which intersects any two of these axes gives rise to intercepts that are irrationally related to one another. Owing to the inequivalence of the axes they are designated by different letters, a, b and c, as illustrated in Figure 145.

**Ground Form.** Before progress can be made in assigning symbols to most of the faces which occur upon the crystal it is necessary arbitrarily to select some one form each of whose faces intersects the three axes, and to agree that the ratios

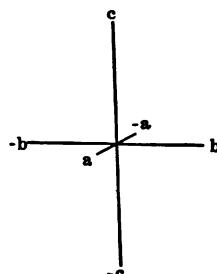


Fig. 145

cut off on these axes by a face of this form shall be the fundamental ratios of the three axes.

This ratio is further simplified by expressing the values  $a$  and  $c$  in terms of  $b$ , which is unity. These values are known as axial ratios and are characteristic for each substance crystallizing in the rhombic system. Since the horizontal section through the ground form is a rhomb with the greater diagonal right and left, it follows that the length of the right and left intercept is always greater than that on the fore and aft axis, so that of the axial ratios  $a$  is always less than  $b$ . In accord with this the fore and aft axis is known as the brachyaxis, while the other lateral axis is the macroaxis. This is sometimes indicated on the letters by signs, thus  $\ddot{a}$ ,  $\ddot{b}$ ,  $\dot{c}$ . For topaz the values are —

$$a: b: c: : .5285: 1: .9539.$$

From these axial ratios the values for the polar elements  $p_0: q_0: r_0$  may be readily obtained by the following transformation:

$$p_0 = \frac{c}{a}; q_0 = c; r_0 = 1.$$

#### Classification of All Planes with Regard to Intersection of Rhombic Axes of Reference.

(a) Plane intersects only one axis:

- (1)  $a$  axis cut — macropinacoid,
- (2)  $b$  axis cut — brachypinacoid,
- (3)  $c$  axis cut — basal pinacoid;

(b) Plane intersects two axes:

- (1)  $a$  axis not cut — brachydome,
- (2)  $b$  axis not cut — macrodome,
- (3)  $c$  axis not cut — rhombic prism;

(c) The plane cuts all three axes — rhombic pyramid.

**Macropinacoid.** This form, like all pinacoids, consists of two planes parallel to one another. It is an open form and can only occur on crystals in combination with other forms. Its symbol is (100).

**Brachypinacoid.** There are here two planes parallel to one another. The centre of symmetry requires these planes to occur in pairs. It is an open form. Symbol (010).

**Basal Pinacoid.** This form is similar to the two forms already described, except that its planes are horizontal and intersect the vertical axis. Symbol (001).

The arrangement for these three pinacoids is indicated in Figure 146, exhibiting the three pinacoids in combination with one another. The pinacoids are invariable and so stand in contrast to the four other types, each of which comprehends a considerable number of distinct forms.

**Brachydome.** A face intersecting the  $b$  and  $c$  axes at finite distances will in this system necessarily be accompanied by three other faces. The brachydome is an open form and cannot occur independently, but is frequently observed in combination with others. The four planes meet in four edges parallel to the  $a$  axis. The brachydome occurs in many different forms according to the ratios at which the  $b$  and  $c$  axes are intersected. The unit brachydome has the symbol (011). The flatter and steeper domes may be indicated by the symbols (0kh) and (0hk) respectively. (Figure 147.)

**Macrodome.** This form in many respects resembles the brachydome, except that its four planes intersect in edges parallel to the  $b$  axis. Each of these planes cuts the  $a$  and  $c$  axes at finite distances. (Figure 148.) The unit macrodome

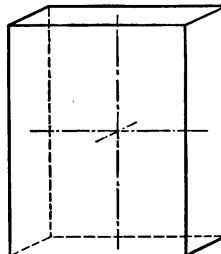


Fig. 146

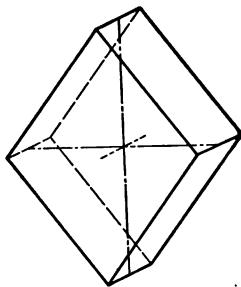


Fig. 147

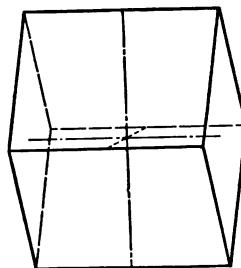


Fig. 148

has the symbol (101), while the flatter and steeper domes have the symbol (k0h) and (h0k) respectively. In the rhombic system, pinacoids and domes are named from the lateral

axis which they do not intersect, thus: the brachydome intersects the macro axis but does not cut the brachy axis.

**Rhombic Prism.** This form closely resembles the two forms just described but differs from them in that the four faces meet in edges parallel to the vertical axis, while each face of the prism intersects the two lateral axes (Figure 149).

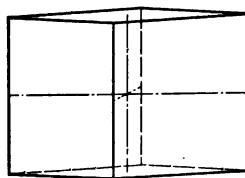


Fig. 149

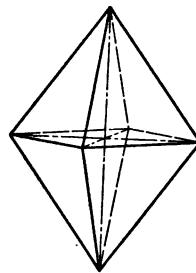


Fig. 150

Symbols (110) for the unit prism and (hk0) and (kh0) are comprehensive symbols for all other prisms.

**Rhombic Pyramid.** The rhombic pyramid is bounded by eight planes each of which is a scalene triangle (Figure 150). Its twelve edges are of three types, four in each set. The six tetrahedral corners are of three pairs, one pair corresponding to the points of exit of each diad axis of symmetry. The unit pyramid known as the ground form intersects the three axes at ratios which become standard for each rhombic substance. The general symbol for the rhombic pyramid is (hkl). There is a series of pyramids intersecting the lateral axes in the same ratio as the ground form but with varying intercepts on the vertical axis. These are known as unit pyramids. Another series, known as the brachypyramids, intersects the a axis at varying lengths while a third series, the macropyramids, intersects the b axis at different lengths. The symbols for these three series are indicated as follows:

Unit pyramids — (hhk) and (khh),

Brachypyramids — (khk) and (hkk),

Macropyramids — (hkh) and (khk).

**Limiting Forms.** Three of the seven forms characteristic of the rhombic system are invariable. These are the pinacoids. Each of the other four types includes a considerable

number of possible forms. This is implied by the use of the letters  $h$ ,  $k$  and  $l$  in writing their symbols.

The pyramid is the most general type in this system because by giving limiting values to the various indices all seven forms of the rhombic system may be derived. When the intercept of the pyramid on the vertical axis becomes infinitely long a prism results, while the basal pinacoid is obtained if the intercept on the vertical axis be infinitely short. Similarly by varying the lengths of the fore and aft axis the series of pyramids culminates in the brachydome and macropinacoid. When the pyramid is given its limiting values on the right and left axis the two forms obtained are the macrodome and the brachypinacoid.

The prisms and domes are also variable forms, each of them swinging between two pinacoidal limits. Thus from the prism the macropinacoid may be obtained by making the intercept on the  $b$  axis infinity and the brachypinacoid by making the intercept on the  $b$  axis zero. In the same way the series of macrodomes swings between the limits indicated by the macropinacoid and the base, while the brachydome finds its limits in the brachypinacoid and the base. These relationships are diagrammatically indicated in Figure 151.

**Calculation of the Ratios  $a:b:c$ .** The value of  $c$  can be most simply calculated from the inner basal angle between the faces  $(011)$  and  $(0\bar{1}\bar{1})$ . Similarly  $a$  may be derived from the angle between the faces  $(110)$  and  $(1\bar{1}0)$ . The information concerning the ratios is complete when we know  $a:b$  and  $b:c$ . While the above represents the simplest calculation it is not the one ordinarily followed. The derivation of the ratios  $a:b:c$  from other angles usually involves spherical trigonometry.

**Gnomonic Projection.** Figure 4 presents a gnomonic projection for some of the forms commonly observed on the mineral topaz. The plane of projection is parallel to the basal pinacoid. It may be observed that the projection points are distributed symmetrically about two lines at right angles

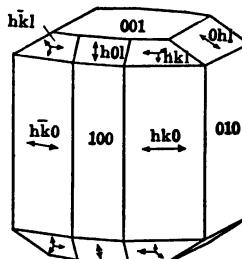


Fig. 151

to one another, and about a central point — the pole of the projection. The two lines of symmetry in a projection correspond to the two planes of symmetry in the rhombic system which stand at right angles to the basal pinacoid, while the centre of symmetry exhibited in the projection corresponds to the diad axis which is normal to the plane of projection. In general the elements of symmetry exhibited in the gnomonic projection are one degree lower than the corresponding elements observed upon the crystals themselves, that is, the planes and axes of symmetry become lines and centres of symmetry in the gnomonic projection.

The radius of the circle on the projection, known as the ground circle, indicates the height of the plane of projection above the centre of the crystal. Since the distance of the various points from one another is dependent upon the height of the plane of projection it is usual to indicate the scale of a projection by drawing a circle whose radius is the height. On measuring the distance of the various projection points from the right and left line of symmetry it will be found that these distances are all simply related to one another. The same relationship holds with regard to the distance of the points from the fore and aft line of symmetry. If we select a fore and aft pace unit  $p_0$  and a right and left pace unit  $q_0$  it will be found that  $p_0$ ,  $q_0$  and  $r_0$  — the height of the plane of projection above the centre as shown by the radius of the ground circle — are for this system all irrationally related to one another. The method by which the symbols and crystallographic constants can be graphically determined from a gnomonic projection has been already explained. In studying the projection of a rhombic crystal, not hitherto described, it is necessary to select arbitrarily the pace lengths  $p_0$  and  $q_0$  after which the symbols for all the faces may be readily indicated and by graphic determination the ratios  $p_0$ ,  $q_0$  and  $r_0$  can be worked out. The transformation of these pace lengths or polar elements into the axial ratios  $a:b:c$  is shown in the following:

$$p_0 = \frac{c}{a}; q_0 = c; r_0 = 1.$$

For the mineral topaz the constants as expressed in the two forms are as follows:

$$a:b:c: : .5285:1: .9539,$$

$$p_0:q_0:r_0: : 1.8049: .9539:1.$$

Figure 14 is a stereographic projection representing for topaz the forms shown in the gnomonic projection.

**Combinations.** In this system six of the seven types are represented by groups of faces which do not enclose space. All rhombic forms, except the pyramids, appear on crystals only in combination with other forms. The faces of the pinacoids do not even intersect to give rise to edges. Figures

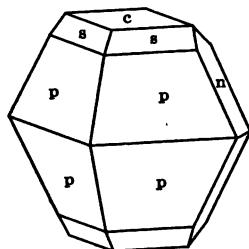


Fig. 152—Sulphur representing the unit pyramid,  $p$ , (111); a second pyramid,  $s$ , (113); the basal pinacoid,  $c$ , (001); and the brachydome,  $n$ , (011).

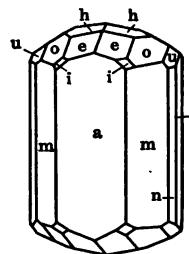


Fig. 153—Hypersthene showing the macropinacoid,  $a$ , (100); the brachyprism,  $b$ , (010); the unit prism,  $m$ , (110); the prism,  $n$ , (120); the brachydome,  $h$ , (014) and four pyramids:  $e$ , (212);  $o$ , (111);  $u$ , (232);  $i$ , (211).

5, 152 and 153 present combinations of rhombic forms occurring on the minerals topaz, sulphur and hypersthene.

**Mineral Examples.** In addition to the minerals already mentioned as crystallizing in this class, the following are important: staurolite, chalcocite and olivine.

#### SPHENOIDAL CLASS

There are two classes of lower symmetry occurring in the rhombic system, one of which is hemihedral, the other hemimorphic. Of the three types of hemihedral selection which have been applied to the earlier systems, only one is capable of producing new geometrical forms in the rhombic system. This selection consists in the extension of those faces occurring in one-half of the octants and the suppression of the faces occurring in the alternate octants. As has been already indicated this type of hemihedrism, known as tetrahedral in

the cubic system, sphenoidal in the tetragonal system and rhombohedral in the hexagonal system, can only give new forms when applied to those holohedra whose faces are confined to a single octant. In the rhombic system this is true for the pyramids only and consequently the only hemihedral type characteristic of this class consists of a pair of

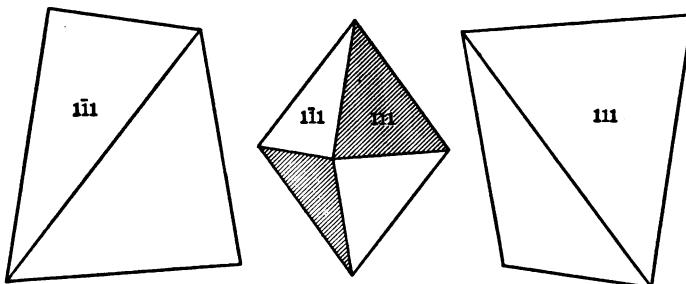


Fig. 154

enantiomorphous forms, one right, the other left. They have a general resemblance to the sphenoids of the tetragonal system. Each form is enclosed by four scalene triangles. Their derivation from the rhombic pyramid is shown in Figure 154. The letter  $\kappa$  indicates this type of hemihedrism while the symbols for the two forms are as follows:

Positive sphenoid —  $\kappa(1\bar{1}1)$ ,  
Negative sphenoid —  $\kappa(1\bar{1}1)$ .

Symmetry about three diad axes at right angles to one another is characteristic of this class.

These forms are usually observed in combination with one or more of the holohedral forms of the rhombic system. Figure 155 shows the occurrence of the right sphenoid in combination with the rhombic prism as exhibited on magnesium sulphate,  $MgSO_4 + 7H_2O$ , and on zinc sulphate,  $ZnSO_4 + 7H_2O$ . Cream of tartar, rochelle salt and tartar emetic also crystallize in this class.

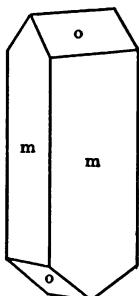


Fig. 155 — Magnesium sulphate,  $MgSO_4 + 7H_2O$ , showing the prism,  $m$ ,  $(110)$  and the positive sphenoid,  $o$ ,  $\kappa(1\bar{1}1)$ . It is not unusual for both the sphenoids to appear on the same crystal, in this way simulating the rhombic pyramid.

When this occurs, however, there is usually a marked difference in central distance between the right and left forms, while the faces on the one form may be rough the other smooth.

#### HEMIMORPHIC CLASS

A number of substances crystallize in such a way as to suggest the holohedral rhombic class when only one end of the crystal is examined but on closer investigation it is found that they are characterized by two planes of symmetry and only one diad axis. The faces intersecting this diad axis at opposite ends are frequently different from one another. These forms are said to be hemimorphic about the diad axis and the axis itself is a hemimorphic axis. It is usual to orient crystals of this class, where only one axis of symmetry is present, so that this axis is vertical. The crystal forms are as follows:

- (a) Rhombic prism, brachypinacoid, macropinacoid, all apparently holohedral,
- (b) Pyramid, macrodome and brachydome and basal pinacoid.

Each of the forms in the latter group becomes two forms in the hemimorphic class which are designated upper and lower,

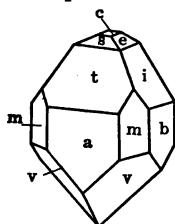


Fig. 156 — Calamine showing the unit prism,  $m$ , (110); the macropinacoid,  $a$ , (100); the brachypinacoid,  $b$ , (010); the half forms of the following: upper basal pinacoid,  $c$ , (001); the macrodomes,  $t$ , (301) and  $s$ , (101); the brachydomes,  $i$ , (031);  $e$ , (011). The lower end of the crystal is enclosed by the half pyramid,  $v$ , (121).

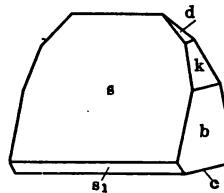


Fig. 157 — Struvite showing the brachypinacoid,  $b$ , (010); the half brachydomes,  $k$ , (041) and  $d$ , (011); the macrodomes,  $s$ , (101) and  $s_1$ , (101); the half basal pinacoid,  $c$ , (001).

while this difference is indicated in their symbols by the sign characteristic of the last index.

The best known example of this class is calamine,  $\text{H}_2\text{Zn}_2\text{SiO}_5$ , which on account of its characteristic development is also known as hemimorphite (Figure 156). Struvite,  $\text{NH}_4\text{MgPO}_4 + 6 \text{H}_2\text{O}$ , also crystallizes in this class (Figure 157). As has already been pointed out hemimorphic crystals are ordinarily pyro-electric. That end of the crystal which becomes negatively electrified with falling temperature is known as the analogue pole, while the opposite end of the hemimorphic axis is known as the antilogue pole.<sup>1</sup>

<sup>1</sup> Some crystallographers use the variable index 1 in the last three systems only for reference to the vertical axis. In this case the relative values for the letters are  $h > k$  while the value of 1 is not fixed. In this work with the exception of Figure 151 the usage of the cubic system has been applied to the last three systems.

## CHAPTER XII

### MONOCLINIC SYSTEM

There are included in the monoclinic system three classes of crystals each of which differs from the others in symmetry. The forms characteristic of each of these classes may be all referred to the same type of axes of reference. Those forms which are characteristic of the classes of lower symmetry may be derived from certain of the forms contained in the holohedral class by regular selection of one-half the faces of some holohedron. Moreover there are certain forms which are common to the various classes contained within this system. The low degree of symmetry which characterizes these classes goes hand in hand as a general rule with molecular complexity. To this system belongs a large proportion of the complex silicates occurring in nature and of the organic products resulting from laboratory experiment.

#### HOLOHEDRAL CLASS

**Symmetry.** Crystals of this class are marked by the presence of one plane of symmetry, a diad axis at right angles to this plane and a centre. Four is the largest number of faces which this low degree of symmetry requires in any monoclinic crystal form. This stands in sharp contrast to the hexoctahedron of the cubic system, whose multiplicity of faces is intimately connected with the unusually high symmetry of the holohedral class of that system.

**Standard Orientation.** It is usual to orient monoclinic crystals in such a way that the diad axis of symmetry is right and left. The one who first describes the crystals of a monoclinic substance orients the crystal so that a prominent series of zone edges is vertical. There is only one other point that must be arbitrarily settled before the orientation

of the monoclinic crystal is fixed. It is necessary to decide which should be regarded as the front of the crystal.

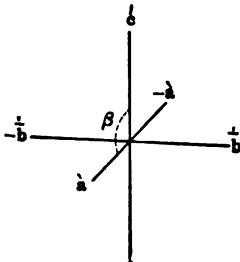


Fig. 158

**Axes of Reference.** Monoclinic forms are referred to three inequivalent axes. The right and left axis  $b$  coincides with the axis of symmetry. The vertical axis  $c$  which is at right angles to  $b$  is usually parallel to some prominent series of zone edges. The fore and aft axis  $a$  is at right angles to  $b$  but is oblique with regard to  $c$ . The direction which the original investigator

selects for the  $a$  axis is usually prominently marked upon the crystal by a series of zone edges fore and aft in their general direction but sloping downward toward the front. Figure 158 shows the relationship of these axes to one another with the letters and signs attached. Figure 159 represents a crystal of orthoclase on which the zone edges selected for the vertical position are indicated by heavy lines, while the edges whose direction was selected for the fore and aft axis  $a$  are indicated by dotted lines. In this system the three axes are designated by terms which have reference to the angles which these axes make with one another.

- (1)  $a$  = clino axis, which is sometimes indicated by an oblique sign above the letter —  $\ddot{a}$ .
- (2)  $b$  = ortho axis, to denote which the sign of perpendicularity is sometimes placed above the letter —  $\dot{b}$ .
- (3)  $c$  = vertical axis, indicated by a vertical sign placed above the letter —  $\dot{c}$ .

In this, as in the rhombic system, the names applied to the axes are of importance in the naming of the various domes and pinacoids characteristic of the system.

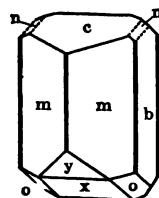


Fig. 159—Orthoclase. The heavy vertical zone lines show the direction chosen to represent the vertical axis,  $c$ ; the dotted lines represent the direction of the oblique axis  $a$  while the axis  $b$  corresponds to the normal to the face  $b$ . Forms shown on the crystal;  $m$ , (110);  $b$ , (010);  $c$ , (001);  $n$ , (021);  $y$ , (201);  $x$ , (101);  $o$ , (111).

**Classification of All Possible Planes with Reference to Intersection of Monoclinic Axes.**

(a) Plane intersects only one axis:

- (1) a axis cut — orthopinacoid,
- (2) b axis cut — clinopinacoid,
- (3) c axis cut — basal pinacoid,

(b) Plane intersects two axes:

- (1) a axis not cut — clinodome,
- (2) c axis not cut — monoclinic prism,
- (3) b axis not cut — two possibilities —

I — Plane lies opposite obtuse angle  $\beta$  — negative hemi-orthodome,

II — Plane lies opposite acute angle  $\beta$  — positive hemi-orthodome,

(c) Plane intersects three axes:

- (1) Plane opposite obtuse angle  $\beta$  — negative hemi-pyramid,
- (2) Plane opposite acute angle  $\beta$  — positive hemi-pyramid.

In all the holohedral classes so far considered there were seven, and only seven, types, but in the monoclinic system there are nine possible types. In the triclinic system, where the symmetry is still lower, the number is further increased.

**Orthopinacoid.** This form consists of two planes parallel to one another and corresponds to the macropinacoid of the rhombic system. The faces intersect the fore and aft axis a and are parallel to the plane containing the b and c axes. Symbol (100).

**Clinopinacoid.** This form is composed of two planes which are parallel to the plane of symmetry. Symbol (010). The corresponding form in the last system is the brachypinacoid.

**Basal Pinacoid.** Like the other two pinacoids this is an open form consisting of a pair of parallel planes. The c axis is intersected at a finite distance, the other axes at infinity. Symbol (001).

Figure 160 shows the three pinacoids in combination on a crystal of pyroxene. It may be noted that these three pinacoids meet to form angles as follows:

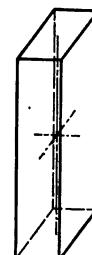


Fig 160

- (100) and (010) — right angles,
- (001) and (010) — right angles,
- (100) and (001) — two acute and two obtuse angles.

**Clinodome.** Forms of this type consist of four planes each intersecting the *b* and *c* axes at finite distances and the *a* axis at infinity. In the monoclinic, as in the rhombic system, some form whose faces intersect all three axes is selected as the ground form and the ratios of the lengths of the intercepts cut off on the three axes are constants characteristic of each crystallized monoclinic substance. For gypsum,  $\text{CaSO}_4 + 2 \text{H}_2\text{O}$ , these values are:

$$a:b:c :: .6895:1: .4133.$$

That clinodome which intersects the *b* and *c* axes in these standard ratios is known as the unit clinodome and has the symbol (011). (0kh) represents in a general way all those clinodomes intersecting the vertical axis at a ratio less than the unit dome. The steeper forms may be represented by (0hk). The edges of intersection of these four planes are parallel to the clino axis. (Figure 161).

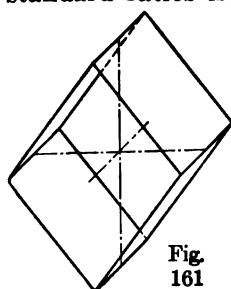


Fig.  
161

**Monoclinic Prism.** The monoclinic prism consists of four planes intersecting the two lateral axes and parallel to the vertical axis. That form which intersects the *a* and *b* axes in the ratios characteristic for the substance is called the unit prism and has the symbol (110). Those prisms intersecting the *b* axis at a relatively greater distance can all be represented by the general symbol (hk0), while those intersecting it at a ratio less than the unit prism are contained in the symbol (kh0). The four zone edges of this prism are parallel to the vertical axis *c*. (Figure 162).

**Negative Hemi-orthodome.** The faces of this form intersect the *a* and *c* axes but are parallel to the *b* axis. It consists of a pair of planes lying opposite the obtuse angles between *a* and *c*. The two faces for the unit form may be symbolized 101 and  $\bar{1}0\bar{1}$ . In addition to the unit form there

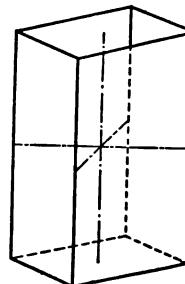


Fig. 162

are steeper and flatter negative hemi-orthodomes which may be symbolized as  $(h0k)$  and  $(k0h)$  respectively. The position of this pair of planes is indicated in Figure 163.

**Positive Hemi-orthodome.** These forms are similar to the ones just described except that the planes lie opposite the

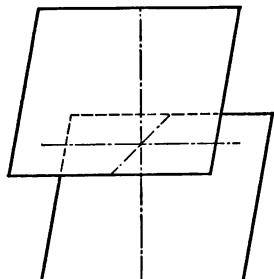


Fig. 163

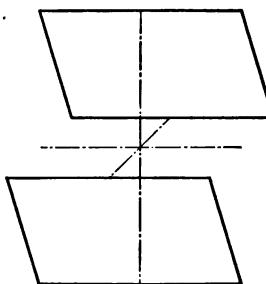


Fig. 164

acute angles between the  $a$  and  $c$  axes. The symbols for the two planes of the unit form are  $\bar{1}01$  and  $10\bar{1}$ . Flatter and steeper positive hemi-orthodomes may be represented by the symbols  $(\bar{k}0h)$  and  $(h0k)$  respectively. (Figure 164).

**Negative Hemi-pyramid.** This form consists of four planes intersecting all three axes and lying opposite the obtuse angle  $\beta$ . Their arrangement is shown in Figure 165. The

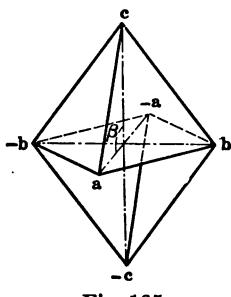


Fig. 165

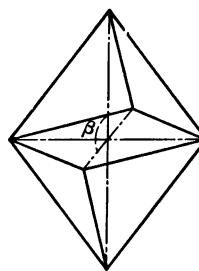


Fig. 166

unit pyramid has the symbol  $(111)$ , while  $(hkl)$  is a general symbol for all negative hemi-pyramids.

**Positive Hemi-pyramid.** This is in all respects similar to the form just described except that its planes lie opposite the acute angle  $\beta$ . General symbol  $(\bar{h}kl)$  (Figure 166).

**Combinations.** In the monoclinic system, as a result of the low degree of symmetry, none of the forms is represented by a sufficient number of faces to enclose space. They are all open forms and can only occur on crystals in combination with one another. Figures 159, 167 and 168 represent crys-

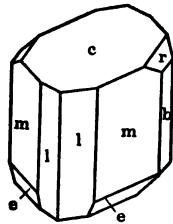


Fig. 167 — Realgar showing the unit prism,  $m$ , (110); the prism,  $l$ , (210); the clinopinacoid,  $b$ , (010); the basal pinacoid,  $c$ , (001); the clinodome,  $r$ , (012); the positive hemipyramid,  $e$ , (111).

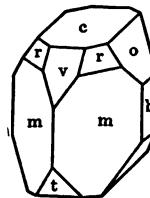


Fig. 168 — Ferrous sulphate,  $\text{FeSO}_4 + 7\text{H}_2\text{O}$ , showing the unit prism,  $m$ , (110); the clinopinacoid,  $b$ , (010); the basal pinacoid,  $c$ , (001); the negative hemiorthodome,  $v$ , (101); the positive hemiorthodome,  $t$ , (101); the clinodome,  $o$ , (011) and the negative hemipyramid,  $r$ , (111).

tals of orthoclase,  $\text{KAlSi}_3\text{O}_8$ , realgar,  $\text{AsS}$ , and ferrous sulphate,  $\text{FeSO}_4 + 6\text{H}_2\text{O}$ , on which these various forms occur in combination.

**Limiting Forms.** The relationship of the forms of the monoclinic system to one another is in most respects similar to that indicated as characteristic of the rhombic system. A pyramid by varying the length of the vertical axis reaches its limits in the basal pinacoid on the one hand and a monoclinic prism on the other. By varying the intercept of the fore and aft axis the pyramids swing between the limits indicated by the orthopinacoid and the clinodome. A similar variation of the right and left axis produces a series of pyramids which find their limits in the clinopinacoid and the orthodome. When we say that the pyramid (hkl) is the most general form in this system we mean thereby that all the other forms of the system may be derived from it merely by giving the limiting values to its various indices. The domes and prisms being variable forms swing between the limit set by a pair of pinacoids, thus, the prism between the orthopinacoid and the clinopinacoid, the orthodomes between the

orthopinacoid and the base and the clinodome between the clinopinacoid and the base.

**Calculation of Monoclinic Ratios  $a:b:c$  and  $\beta$ .** In this, as in the rhombic system, the ratios of the  $a$  and  $c$  axes are expressed in terms of  $b$  which is unity. The angle between the  $a$  and  $c$  axes is oblique in this system and varies considerably in different monoclinic substances. This angle is indicated by the Greek letter  $\beta$ . It is necessary, therefore, to determine for monoclinic crystals the magnitude of the angle  $\beta$  in addition to the axial ratios  $a:b:c$ .

The intersections of the  $a$  and  $c$  axes give rise to two oblique angles, one greater and the other less than  $90^\circ$ . It is usual to employ the letter  $\beta$  to indicate the smaller of these. This angle may be determined by direct measurement of the angle between the orthopinacoid and the base.

**Gnomonic Projection.** Figure 169 is a gnomonic projection representing the forms most commonly observed on epidote. A crystal representing all the forms projected is illustrated in Figure 170. The plane of projection is at right angles to the vertical axis. The distribution of the projection points is symmetrical with regard to one line which represents the trace of the plane of symmetry. For the monoclinic system the centre or pole of the projection is not a projection point for any crystal face. In this regard the monoclinic projection differs from those of the four previous systems. There are numerous zone lines, some right and left, others fore and aft. These two chief series of zone lines intersect at right angles. Except the line of symmetry, which usually represents a prominent zone, no other zone line passes through the pole of projection.

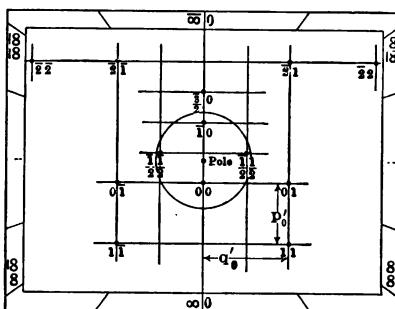


Fig. 169

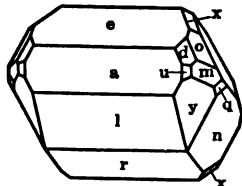
The measurement to the right and left of the line of symmetry is made in terms of paces designated  $q'_0$  and all the distances to the right or left of that line are simply and rationally re-

lated to one another. It will be observed that a fore and aft pace may be employed to measure conveniently the distance between any pair of the right and left zone lines. Such a distance is  $p'_0$ . If a new monoclinic crystal were being described

the pace lengths  $p'_0$  and  $q'_0$  might be arbitrarily selected from the projection, but in dealing with crystals already described, it is necessary to determine the symbols for one or two faces so as to fix the pace lengths  $p'_0$  and  $q'_0$  before proceeding to indicate the symbols for all the other faces or to graphically determine by a system of averages the values for  $p'_0$  and  $q'_0$ . Since in this system no right and left zone line passes through the polar point it becomes necessary to select some other zone line from which the measurements forward and backward are to be made. Usually the first prominent right and left zone line lying in front of the polar point is the

Fig. 170—Epidote showing the orthopinacoid, *a*, (100); the negative hemiorthodome, *e*, (101); the positive hemiorthodomes, *l*, (201) and *r*, (101); the prism, *m*, (110); the prism, *u*, (210); the clinodome, *o*, (011); the negative hemipyramid, *d*, (111); four positive hemipyramids, *n*, (111); *x*, (112) *q*, (221) and *y*, (211).

line of reference from which forward and backward measurements are made. This line passes through the projection points for the following faces: (001), (011) and (011) and at infinity through (010) and (010). Having selected the line from which fore and aft measurements are to be made and the pace lengths  $p'_0$  and  $q'_0$  the symbols may then be assigned to all the faces and the polar elements  $p_0$  and  $q_0$  determined. The obliquity between the *a* and *c* axes is indicated in each projection by the distance between the pole and the projection for the face (001). This value is designated *e'*, which is equal to the tangent of  $\rho$  for (001). The values  $p'_0$  and  $q'_0$  and *e'* as measured from the projection are expressed in terms of the height of plane of projection  $h'_0$ . It is the custom of crystallographers to express these values in terms of the length of the normal of (001) as measured from the crystal centre to its projection point. This may be done by multiplying these values by  $\sin. \beta$ . The ratios so obtained are  $p_0$ ,  $q_0$ ,  $h_0$  and *e*.



$$q_0 = c \cdot \sin. \beta; p_0 = \frac{c}{a}$$

## HEMIHEDRAL CLASS

It is possible to select one-half of the four faces of the hemipyramid of this system in three ways as illustrated in Figure 165. The selection of the faces  $111$  and  $\bar{1}\bar{1}1$  or  $1\bar{1}1$  and  $\bar{1}\bar{1}\bar{1}$  would give us a form composed of two faces parallel to one another and symmetrical about the centre only. This is equivalent to the holohedral triclinic class and will be considered in connection with the triclinic system. The second selection of one-half the four planes takes place in such a way that the plane of symmetry is preserved, the centre and diad axes having disappeared. The extension of either  $111$  and  $\bar{1}\bar{1}1$  or of  $1\bar{1}1$  and  $\bar{1}\bar{1}\bar{1}$  gives rise to this type of symmetry. There are two congruent forms from each of the monoclinic holohedra, except the clinopinacoid. Pyroxene which belongs to this class is represented in Figure 171.

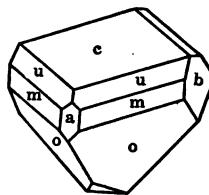


Fig. 171 — Pyroxene showing hemihedral development; forms present — the orthopinacoid,  $a$ ,  $(100)$ ; the clinopinacoid,  $b$ ,  $(010)$ ; the prism,  $m$ ,  $(110)$ ; half of the faces of the hemipyramids  $o$  and  $u$ ; the basal pinacoid  $c$ .

## HEMIMORPHIC CLASS

The forms characteristic of this class may be derived from various holohedra by the selection of those parts of a crystal form on one side of the plane of symmetry. Here the diad axis is the only element of symmetry. This axis is hemimorphic since the faces intersecting one end of this axis are not necessarily accompanied by similar faces at the opposite end. Those holohedral forms whose faces are parallel to the  $b$  axis are represented by forms geometrically identical with those of the holohedral class. On the other hand all holohedral forms which intersect the  $b$  axis at a finite distance are here represented by pairs of hemimorphic forms either of which may occur independently of the other. The forms peculiar to this class are as follows:

1. Negative tetra-pyramids — right ( $hkl$ ) and left ( $h\bar{k}l$ ),
2. Positive tetra-pyramids — right ( $\bar{h}kl$ ) and left ( $h\bar{k}l$ ),
3. Hemi-prisms — right ( $hk0$ ) and left ( $h\bar{k}0$ ),
4. Hemi-clinodomes — right ( $0hk$ ) and left ( $0\bar{h}k$ ),
5. Clinopinacoids — right ( $010$ ) and left ( $0\bar{1}0$ ).

Two is the maximum number of faces belonging to a crystal form in this class, while for the hemi-clinopinacoid the form is composed of a single face.

Tartaric acid,  $C_4H_6O_6$ , quercite,  $C_6H_{12}O_6$ , (Figure 172) and cane sugar crystallize in the hemimorphic class. If

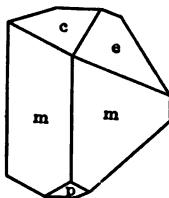


Fig. 172 — Quercite exhibiting hemimorphism on the axis of symmetry. Forms present: the prism,  $m$ , ( $110$ ); the basal pinacoid,  $c$ , ( $001$ ); the positive hemiorthodome,  $p$ , ( $\bar{1}01$ ) and the right hemiorthodome,  $e$ , ( $011$ ).

however, a solution be prepared from equal quantities of substances representing the two types the resulting solution is

sodium ammonium racemate be allowed to crystallize from a solution of water, large crystals are produced similar to those of tartaric acid which are represented in Figure 173. It will be observed that the crystals here represented are similar to one another and are distinguished by the right and left handed arrangement of the faces. The one is the mirror image of the other. If a solution be prepared exclusively from crystals of the one class it is found to turn the plane of polarization to the right, while a similar solution of the other turns it in the opposite direction. If,

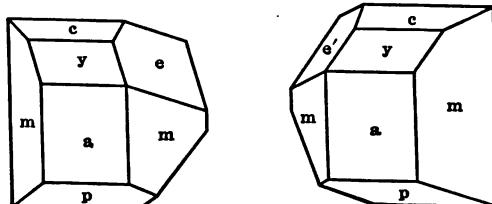


Fig. 173 — Tartaric acid showing right and left handed enantiomorphous forms. The forms present are — the orthopinacoid,  $a$ , ( $100$ ); the basal pinacoid,  $c$ , ( $001$ ); the unit prism,  $m$ , ( $110$ ); the negative hemiorthodome,  $y$ , ( $101$ ); the positive hemiorthodome,  $p$ , ( $\bar{1}01$ ). On the first crystal the hemi-clinodome,  $e$ , ( $011$ ), appears while on the second the corresponding form  $e'$ , ( $0\bar{1}1$ ), is present.

inactive with regard to polarized light. It was Pasteur who first demonstrated the relationship between geometrically right handed and left handed crystals and dextro-rotary and lavo-rotary substances. Racemic acid is a mixture of equal proportions of two distinct tartaric acids possessing the same chemical composition and individually giving rise to crystals showing this right-handed and left-handed character. Lavo-rotary and dextro-rotary organic substances are known as stereoisomers. Substances crystallizing in this class, being characterized by a hemi-morphic axis of symmetry, show pyro-electricity as a result of change of temperature.

## CHAPTER XIII

### TRICLINIC SYSTEM

Only two classes of crystals are included in the triclinic system. One of these, the holohedral triclinic class, is characterized by symmetry about a centre. The other class, which is hemihedral triclinic, is marked by the absence of all symmetry. In the latter class any face may appear without the compulsory appearance at the same time of any other face. The crystal form consists of only one face.

#### HOLOHEDRAL CLASS

**Symmetry.** The symmetrical arrangement of faces in pairs about a centre is peculiar to the holohedral triclinic class.

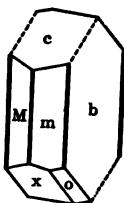


Fig. 174 — Albite — The heavy lines indicate the vertical zone edges of the crystal which correspond to the vertical axis  $c$ ; the dotted lines similarly correspond to the fore and aft axis  $a$  while the long right and left lines show the direction chosen for the right and left axis,  $b$ . Forms present — the basal pinacoid,  $c$ , (001); the brachypinacoid,  $b$ , (010); the right hemiprism,  $M$ , (110); the left hemiprism,  $m$ , (110); the tetra-pyramid,  $o$ , (111) and the hemimacrodome,  $x$ , (403).

The various crystal forms are composed of pairs of faces so that in a way they may all be regarded as pinacoidal. In connection with this fact it may be observed that this class is sometimes referred to as triclinic pinacoidal in contrast to the class without symmetry, which is known as the triclinic asymmetric class.

**Standard Orientation.** Very great latitude is permitted to the original investigator in the case of triclinic crystals. As there are no planes or axes of symmetry there is no very definite regulation as to the orientation of such crystals. In triclinic

crystals of both classes there is a complete absence of rectangularity, no plane or edge being at right angles to any other plane or edge. In selecting the standard orientation of a new triclinic substance it is usual to pick out three sets of zone edges intersecting one another approximately at right angles, and to so orient the crystal that these three edge directions are approximately fore and aft, right and left and vertical. Figure 174 exhibits a crystal of albite with the three edge directions shown in lines of different type.

**Axes of Reference.** The faces of triclinic forms are referred to three inequivalent axes parallel to the three series of zone edges selected as a basis for orientation. As indicated in Figure 175 the crystal is usually so oriented that the obtuse angles formed by the three axes are all enclosed by the positive ends of the axes. The terminology in regard to the axes of reference for the triclinic system is similar to that employed for the rhombic system. The axes  $a$ ,  $b$  and  $c$  are designated brachy, macro and vertical axes, respectively.

Since the angles between the various pairs of axes are always oblique in this system it is convenient to indicate the magnitude of those angles by means of the three letters  $\alpha$ ,  $\beta$  and  $\gamma$ .

$\alpha$  is the angle between the  $b$  and  $c$  axes,

$\beta$  is the angle between the  $a$  and  $c$  axes,

$\gamma$  is the angle between the  $a$  and  $b$  axes.

The letter  $\beta$  was used in the monoclinic system to indicate the value of the angle between the  $a$  and  $c$  axes.

#### Classification of All Possible Planes Intersecting Triclinic Axes of Reference.

(a) The plane intersects only one axis:

- (1)  $a$  axis cut — macropinacoid,
- (2)  $b$  axis cut — brachypinacoid,
- (3)  $c$  axis cut — basal pinacoid

(b) The plane intersects two axes:

- (1)  $a$  axis not cut — two possibilities —

1st. Plane opposite obtuse angle  $\alpha$  — right hemi-brachydome,  
2nd. Plane opposite acute angle  $\alpha$  — left hemi-brachydome,

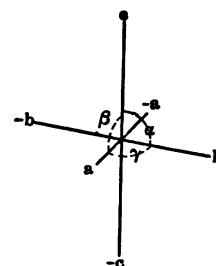


Fig. 175

(2) *b* axis not cut — two possibilities —  
 1st. Plane opposite obtuse angle  $\beta$  — upper hemi-macrodome,  
 2nd. Plane opposite acute angle  $\beta$  — lower hemi-macrodome,

(3) *c* axis not cut — two possibilities —  
 1st. Plane opposite obtuse angle  $\gamma$  — right hemi-prism,  
 2nd. Plane opposite acute angle  $\gamma$  — left hemi-prism,

(c) The plane intersects three axes — four possibilities:

1st. Right upper tetra-pyramid,  
 2nd. Left upper tetra-pyramid,  
 3rd. Right lower tetra-pyramid,  
 4th. Left lower tetra-pyramid.

There are, therefore, in the triclinic system, owing to the low degree of symmetry, thirteen distinct ways in which planes may intersect the axes of reference.

**Tetra-pyramids.** The forms belonging to each of these four types are composed of pairs of planes parallel to one another occurring on opposite sides of a crystal. The rhombic pyramid, which is bounded by eight planes, corresponds to the eight planes on the positive and negative hemi-pyramids of

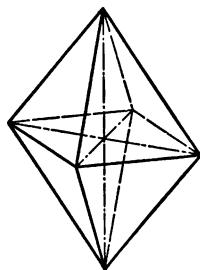


Fig. 176

the monoclinic system and to the eight planes of the four tetra-pyramids of the triclinic system. These four forms when occurring together enclose space giving rise to a combination as represented in Figure 176. When the original investigator has selected a unit tetra-pyramid he incidentally settles the ratios of the lengths of the intercepts on the three axes of reference, and after this arbitrary decision has been reached the six crystallographic constants of the triclinic system are fixed. These characteristic constants for the triclinic mineral albite,  $\text{NaAlSi}_3\text{O}_8$ , are as follows:

$$a : b : c : .6335 : 1 : .5577,$$

$$\alpha = 94^\circ 3' : \beta = 116^\circ 29' : \gamma = 88^\circ 9'.$$

The symbols for the unit forms, as well as the general symbol for each of the four tetra-pyramids, are as follows:

Right upper tetra-pyramid — (111) and (hkl),  
 Left upper tetra-pyramid — (1̄1̄1̄) and (h̄kl̄),  
 Right lower tetra-pyramid — (111̄) and (hkl̄),  
 Left lower tetra-pyramid — (1̄1̄1̄) and (h̄kl̄).

**Hemi-macrodomes.** There are in this system two hemi-macrodomes, each composed of a pair of faces corresponding to the pair of hemi-orthodomes characteristic of the last system. Symbols:

Upper hemi-macrodome, unit form (101),

general symbol (h0k),

Lower hemi-macrodome, unit form (10 $\bar{1}$ ),

general symbol (h0 $\bar{k}$ ).

**Hemi-brachydomes.** These forms are similar in their occurrence to the pair of forms just described. Symbols:

Right hemi-brachydome — unit form (011) —

general symbol (0hk),

Left hemi-brachydome — unit form (0 $\bar{1}$ 1) —

general symbol (0 $\bar{h}$ k).

**Hemi-prisms.** The relation of these prisms to one another is the same as that which maintains between the pair of brachydomes. Symbols:

Right hemi-prism — unit form (110) —

general symbol (hk0),

Left hemi-prism — unit form (1 $\bar{1}$ 0) —

general symbol (h $\bar{k}$ 0).

**Pinacoids.** The three pinacoids bear the same relationship to one another as do the three pinacoids in the last two systems. Their mode of intersection when in combination with one another is shown in Figure 177. These three forms are invariable in contrast to those types already considered, where, in addition to the unit form, a large number of other forms are possible, such as have been indicated by the use of the general symbols. Symbols:

Macropinacoid (100),

Brachypinacoid (010),

Basal pinacoid (001).

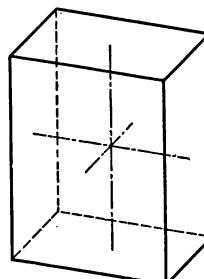


Fig. 177

**Combinations.** In this class where each crystal form is made up of only two faces there are no closed forms, so that every triclinic crystal must present two or more distinct forms. Figures 178, 174, 179 and 180 show the combinations of triclinic forms observed on crystals of anorthite, albite,

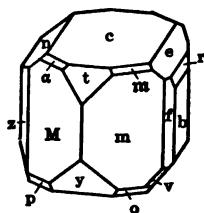


Fig. 178—Anorthite exhibiting the basal pinacoid,  $c$ , (001); the brachypinacoid  $b$ , (010); the right hemiprism,  $m$ , (110); the left hemiprism,  $M$ , (110); the right hemiprism,  $f$ , (130); the left hemiprism,  $z$ , (130); the right hemibrachydome,  $e$ , (021) and,  $r$ , (061); the left hemibrachydome,  $n$ , (021); and the tetrapyrramids,  $a$ ,  $m$ ,  $p$ ,  $o$ , and  $v$ .

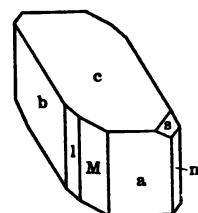


Fig. 179—Copper sulphate,  $\text{CuSO}_4 + 5 \text{H}_2\text{O}$ , showing the basal pinacoid,  $c$ , (001); the brachypinacoid,  $b$ , (010); the macropinacoid,  $a$ , (100); the left hemiprism,  $M$ , (110); the left hemiprism,  $l$ , (120); the right hemiprism,  $m$ , (110) and the right front upper tetrapyrramid,  $s$ , (111).

copper sulphate,  $\text{CuSO}_4 + 5 \text{H}_2\text{O}$ , and calcium hyposulphite,  $\text{CaS}_2\text{O}_3 + 6 \text{H}_2\text{O}$ , respectively.

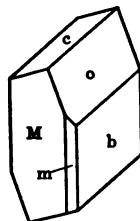


Fig. 180—Calcium hyposulphite showing the basal pinacoid,  $c$ , (001); the brachypinacoid,  $b$ , (010); the right hemiprism,  $m$ , (110); the left hemiprism,  $M$ , (110) and the right hemibrachydome,  $o$ , (011).

**Limiting Forms.** In this system the pyramid is the most general form and by varying the intercept on the vertical axis a series of pyramids is obtained which finds its limits in the basal pinacoid and prism. Similarly by varying the intercept on the macroaxis various pyramids are obtained limited by the brachypinacoid and macrodome. In the same way by varying the intercept on the brachy axis we obtain a series of pyramids whose limits are the macropinacoid and the brachydome. In this system, as in the last, variations of the domes and prisms reach their limit in pairs of pinacoids.

**Projections.** Figure 181 represents a gnomonic projection for the crystal of anorthite, shown in Figure 178. It may be observed that in this projection the points occur on zone lines but there is no symmetrical arrangement about any line or point. This corresponds to the known symmetry of the triclinic holohedral class. The zone lines do not intersect

at right angles, which is in accord with the general obliquity characteristic of the system. The pole of the projection is not the projection point for any face, neither does any zone line pass through the pole. The fore and aft pace length  $p'_0$  is indicated by half the distance between the points for 11 and  $\bar{1}1$ , while the right and left pace  $q'_0$  corresponds to half the distance between 11 and  $1\bar{1}$ , and  $h'_0$  equals radius of the

ground circle. From these lengths, which may be measured on a gnomonic projection, the polar elements  $p_0: q_0: r_0$  may be derived. Owing to the obliquity of all the axes of reference crystallographic calculations in connection with the triclinic system are somewhat more difficult than those of the other systems.<sup>1</sup>

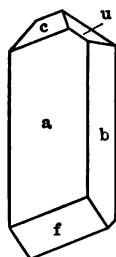


Fig. 182 — Acid strontium tartate. Hemihedral triclinic, exhibiting the half basal pinacoid,  $c$ , (001); the half brachypinacoid,  $b$ , (010); the half macropinacoid,  $a$ , (100); the pyramid,  $u$ , ( $\bar{1}22$ ) and the lower macrodome,  $f$ , (101). Each of these crystal forms consists of a single face.

composed of a single plane. As a matter of fact on crystals of the few substances belonging to this class faces frequently occur in pairs but their opposite and parallel faces are physically distinct forms and their simultaneous occurrence is more or

<sup>1</sup> For an example of calculations for this system see Goldschmidt and Borgstrom, "Zeitschrift für Krystallographie." Band XLI.

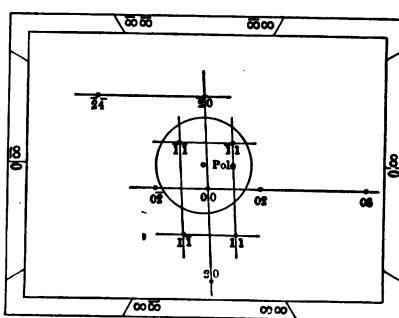


Fig. 181

#### HEMIHEDRAL CLASS

In this class are comprised those crystals whose distribution of faces is such as to indicate the entire absence of symmetry. Any face which is in accord with the rationality of axial parameters may occur unaccompanied by any other face. The crystal form is here

less accidental. The substances belonging to this class are all chemically complex so that they stand in a sharp contrast to most of the substances crystallizing in the cubic system where the degree of symmetry is very high. They are all products of the chemical laboratory. Figure 182 indicates the form of crystals of strontium bitartrate,  $\text{Sr}(\text{C}_4\text{H}_4\text{O}_6\text{H})_2 + 4 \text{H}_2\text{O}$ .

## CHAPTER XIV

### CRYSTAL AGGREGATES

#### PARALLEL GROWTHS

There is a marked tendency for several crystals whose corresponding directions are all parallel to one another to grow together into one mass, while the individuality of the various crystals is shown on certain portions of the surface by the occurrence of the faces characteristic of each. This is often the case with the mineral quartz (Figure 183). This method of aggregation is described as a parallel growth.

Crystals of different substances belonging to the same crystal system sometimes are grown together in such a way that the directions of the axes of reference are common to both, certain edges and planes of the one crystal being parallel to the corresponding edges and planes of the other. In this case, although the substances are different from one another, a complete parallelism in orientation maintains. This is shown with remarkable perfection in Figure 184, exhibiting a crystal of zircon enclosed in a crystal of xenotime. Both these minerals are of the tetragonal system and they are so intergrown that the corresponding tetragonal directions of the two substances are parallel.

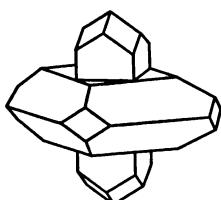


Fig. 183 — Parallel intergrowth of zircon and xenotime.

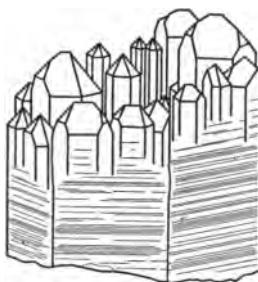


Fig. 183 — Quartz showing parallel growth.

A parallelism less perfect is sometimes exhibited by substances belonging to different crystal systems where certain planes and edges are common to the two crystals. This has

been frequently observed on crystals of staurolite and kyanite from the St. Gothard region in Switzerland. The former of these minerals is rhombic, the latter triclinic.

#### IRREGULAR AGGREGATES

In rocks containing several prominent minerals there is usually no perceptible regularity in the arrangement of the crystals with regard to one another. Where the crystal individuals have grown and developed until they come into contact with other individuals there is a general absence of crystal faces, the boundaries between the individuals being quite irregular. This lack of direction in the orientation of the crystal units is exceedingly common in rocks and is sometimes referred to as irregular aggregation.

#### FIBROUS RADIATING AGGREGATES

Those minerals characterized by acicular crystals are often found in more or less spherical or hemispherical masses built up of needles radiating from a centre. This method of aggregation is very characteristic for natrolite, wavellite and hematite.

#### TWINNING

On simple crystals such as have been described in connection with the various systems there is a general absence of re-entrant angles and it has been assumed that all the particles of which a crystal is built up are absolutely parallel to one another. However, we frequently find crystals characterized by re-entrant angles which may show cleavage in different directions in different parts of what is apparently the same crystal. Such crystals are said to be *twins* and are made up of two or more crystal individuals grown together in some regular way. A very simple case of this kind is exhibited by the *swallow-tailed twin* of gypsum (Figure 185). If this twin of gypsum be divided into two parts by the plane marked by the dotted lines, and one of the parts be revolved  $180^\circ$  about a line normal to the dividing plane, the two parts in their new position will appear

to represent only an ordinary crystal of this substance. To explain forms of this type an hypothesis has been suggested according to which early during the growth of the

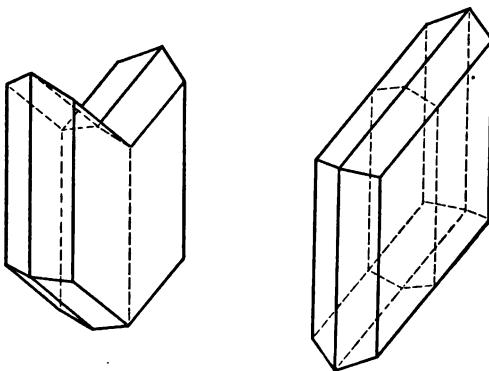


Fig. 185 — Showing the derivation of the swallow-tailed twin of gypsum from the simple crystal individual by dividing the latter by a plane parallel to the orthopinacoid and then revolving one half about a line normal to this plane.

crystal two crystal particles as represented in Figure 186 were brought together in such a way as to secure parallelism along two directions, while the third directions are not parallel. From this partial parallelism of the two crystal building particles, the crystal in its complex form has been produced — each particle attracting to itself and orienting in its own fashion other particles of the same substance. As

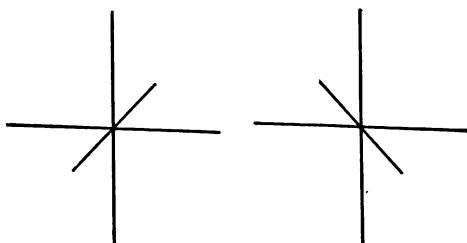


Fig. 186 — Diagram showing partial parallelism between two particles.

the two original particles were symmetrical with regard to the plane between them, so are the resultant half crystals symmetrical about this same plane. The plane about which

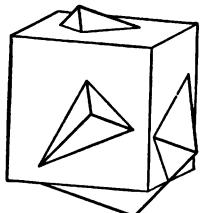
the two halves of the twin crystals are symmetrically arranged is always a possible crystal face of the substance in question, that is, the direction of the *twinning plane*, as it is called, is such as to intersect the axes of reference in accordance with the law of rationality of axial parameters. The twinning plane obviously can never have the direction of a plane of symmetry on the simple crystal. Twin crystals, such as that of gypsum, present a higher degree of symmetry than the individual crystals from which they are derived. The *swallow-tailed* twin of gypsum (Figure 185) is characterized by the presence of two planes of symmetry and one diad axis of symmetry, simulating in this way a higher grade of symmetry. This is a general effect of twinning.

The normal to the twinning plane about which the half of the twin may be revolved so as to become parallel to the other is known as the *twinning axis*. Twin crystals may be supposed to be produced by placing two crystal individuals of the same substance parallel to one another and revolving one of them  $180^\circ$  about its twinning axis. The plane along which these two individuals in their new orientations are brought into contact with one another is known as the *composition face*. Usually the composition face is identical with the twinning plane but in certain types of twinning for orthoclase the composition face and twinning plane are not identical.

The statement of the directions of the twinning plane composition face and twinning axis for any particular substance is known as the *twinning law* for that substance. Certain substances such as orthoclase twin according to several distinct laws.

Fig. 187—Fluorite exhibiting interpenetration twinning of two cubes with the twinning axis normal to the face of the octahedron.

Two Chief Types of Twinning. In the commonest form of twinning illustrated by Figure 185 the two individuals are merely brought into contact with one another. There is a surface more or less regular which divides the twin crystal into two such parts that all the particles on each side of that surface have a common orientation. This mode of aggregation gives rise to *contact twins*. Another type of



twinning is illustrated by Figure 187, representing two cubes of fluorite, so intergrown as to have a common centre. In this case there is no surface dividing particles of the two orientations from one another. Throughout the central part of the twin crystal the portions representing the different orientations are irregularly intermingled. This type of intergrowth, in which the general outline of the one crystal includes a large proportion of the substances contained within the general

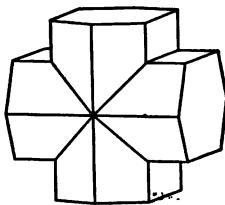


Fig. 188 — Interpenetration twin of staurolite—twinning plane the brachydome (032).

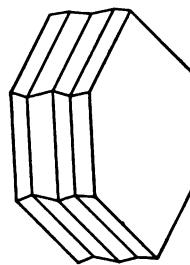


Fig. 189 — Albite showing repeated contact twinning according to the albite law.

outline of the other, gives rise to *penetration twins*. Figure 188 indicates this type for staurolite.

**Repeated Twinning.** The process which gives rise to two distinct individuals showing partial parallelism is often repeated as in the case of albite. Figure 189 represents the common twinning of albite, where the two individuals are placed side by side, with a brachypinacoid as the composition face. In albite this process is repeated indefinitely so that the twin crystal is composed of a large number of thin plates placed side by side in such a way that the 1st, 3rd, 5th, 7th, etc., are absolutely parallel to one another, while the 2nd, 4th, 6th and 8th have a similar relationship to one another. This is a case of repeated twinning in which the alternate individuals are parallel. In nature there is a marked tendency for the individual in such repeated twins to be shortened in a direction at right angles to the composition face. Each of these twin plates is a lamella and this type of repeated twinning is known as *lamellar* or *polysynthetic twinning*. Aragonite frequently twins in this fashion.

A second type of repeated twinning results from the twinning of the successive individuals according to the same twinning law except that the twinning plane for any pair of individuals is not the same as that for any preceding pair. The twinning plane is another face of the same crystal form.

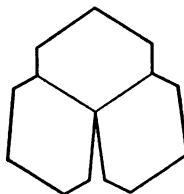


Fig. 190 — Cyclic contact twinning.

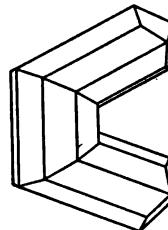


Fig. 191 — Cyclic contact twinning as shown on rutile — twinning plane a face of the pyramid of the second order (101).

Here the alternate individuals are not parallel to one another. Figure 190 represents diagrammatically a cross-section through such a series of twins. This type of repeated twinning tends to arrange the successive members in a circular fashion and is known as *cyclic twinning*. Figure 191 represents this type of twinning as applied to the mineral rutile. It is proposed in the following pages to consider the chief twinning laws of the different crystal systems and to illustrate these laws by reference to mineral examples.

### COMMON TWINNING LAWS

#### CUBIC SYSTEM

**Spinel,  $MgAl_2O_4$ .** The commonest twinning law of the cubic system is that frequently observed on spinel, magnetite, native gold and native copper. This is represented in Figure 192. By the spinel law we obtain contact twins whose twinning plane and composition face is (111).

**Fluorite,  $CaF_2$ .** In fluorite (Figure 187) the twinning plane is also (111) the same as for spinel, but this mineral gives rise to penetration twins. If one of the individuals shown be revolved  $180^\circ$  about the triad axis it will then

occupy such a position that all its edges and faces are parallel to the corresponding edges and faces of the other individual.

**Pyrite**,  $\text{FeS}_2$ . The very beautiful twin of pyrite in Figure 193 shows penetration twinning with (110) as the twinning plane. This peculiar twin is known as the *iron cross*. It should be noted that as a result of twinning the pyrite has attained a symmetry much higher than that characteristic for the individual crystal. Twinning which gives to hemihedral crystals the symmetry characteristic of the holohedral class of the system to which they belong is known

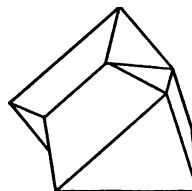


Fig. 192 — Contact twinning of spinel — twinning plane the octahedron (111).

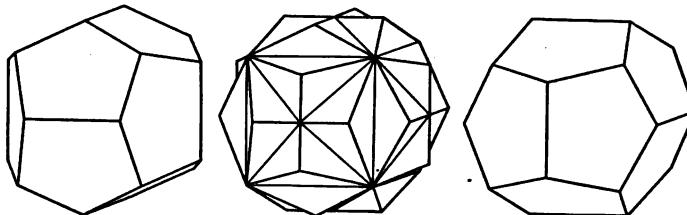


Fig. 193 — The Iron Cross twin of pyrite — twinning plane (110).

as *supplementary* twinning. The iron cross twin of pyrite is a good example of this type.

#### TETRAGONAL SYSTEM

**Cassiterite**,  $\text{SnO}_2$ . This tetragonal mineral very seldom occurs in single crystals. The twinning of cassiterite may be described as follows: contact twins with the pyramid of the second order (101) as twinning plane and composition face (Figure 194). Since the two parts of the crystal are bent towards one another in a way suggesting a knee the cassiterite twin is sometimes spoken of as a *geniculate twin*. Rutile,  $\text{TiO}_2$ , is frequently twinned according to the same law (Figure 191) while in the closely related mineral zircon,  $\text{ZrSiO}_4$ , twinning according to this law is quite rare. The crystals of the mineral rutile commonly twin with the pyramid of the second order as twinning plane and composition

face, but in such a fashion that while the twinning is repeated the alternate individuals are not parallel. For the second and third individuals the twinning plane while still a plane

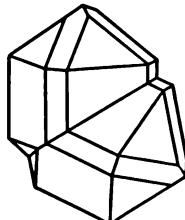


Fig. 194 — Geniculate twinning of cassiterite — twinning plane (101).

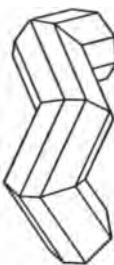


Fig. 195 — Cyclic twinning of rutile. The twinning planes are faces of the pyramid (101).



Fig. 196 — Complete cycle resulting from twinning shown in Figure 191.

of the pyramid of the second order is not the *same* plane as that which served as twinning plane for the first and second individuals. Figure 195 shows a repeated twin of this type composed of four individuals. If the process of cyclical twinning be continued an aggregate such as Figure 196 may result.

#### HEXAGONAL SYSTEM

**Calcite,  $\text{CaCO}_3$ .** Calcite exhibits polysynthetic twinning in which the twinning plane and composition face are parallel

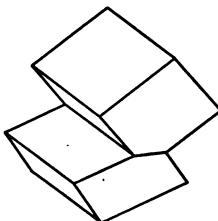


Fig. 197 — Calcite showing in its simplest form twinning with the rhombohedron of the first order  $\kappa(01\bar{1}2)$  as twinning plane.

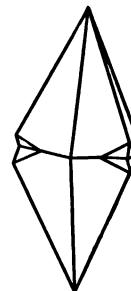


Fig. 198 — Contact twinning in calcite with the basal pinacoid (0001) as twinning plane.

to the negative rhombohedron —  $\kappa(01\bar{1}2)$ . Figure 197 illustrates the relative positions of the first pair in twinning according to this law.

Another method of twinning exhibited by this mineral is shown in Figure 198. For this form the twinning plane and composition face is the basal pinacoid (0001) which for calcite is not a plane of symmetry.

The butterfly twin of calcite results from twinning on a steep, negative rhombohedron —  $\kappa(02\bar{2}1)$ .

**Quartz,  $\text{SiO}_2$ .** Tetartohedral crystals of quartz frequently give rise to penetration twins — twinning plane, the basal

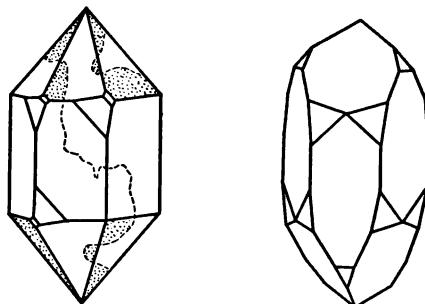


Fig. 199 — Types resulting from twinning of tetartohedral crystals of quartz — the first by the interpenetration of two right handed individuals with (0001) as twinning plane — the second by the interpenetration of right and left hand individuals with the same twinning plane.

pinacoid (0001). The first picture in Figure 199 represents the twinning of two right-handed quartz crystals. Twinning of crystals of opposite types give rise to the twin illustrated in the second picture (Figure 199). The first type of twinning in the case of quartz is by the Dauphiné law, while the second is according to the Brazilian law. By means of physical tests it is sometimes possible to select the faces or parts of faces belonging to each of the twinned individuals.

#### RHOMBIC SYSTEM

**Aragonite,  $\text{CaCO}_3$ .** For this mineral the common twinning plane is the unit prism (110). It is usually repeated and may be polysynthetic or cyclic (Figure 190). Frequently three individuals with the orientation of cyclic twins inter-

penetrate so as to give rise to pseudo-hexagonal forms. This approximation as a result of twinning of crystals of a lower degree of symmetry to forms of a higher degree of symmetry is known as *mimicry*. In this case the pseudo-hexagonal form is due to the fact that the prism angle for aragonite is nearly  $60^{\circ}$ . The mineral cordierite produces interpenetrating twins similar to those of aragonite.

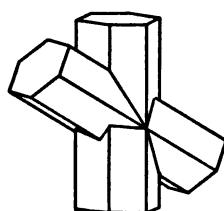


Fig. 200 — Interpenetration twin of staurolite — twinning plane the pyramid (232).

to an oblique cross (Figure 200). The twinning plane for this second type is the pyramid (232).

#### MONOCLINIC SYSTEM

**Gypsum**,  $\text{CaSO}_4 + 2 \text{H}_2\text{O}$ . The swallow-tailed twin of gypsum (Figure 185) results from contact twinning when the orthopinacoid (100) serves as twinning plane and composition face. Many monoclinic minerals twin in this fashion, among which may be mentioned augite (Figure 201), hornblende, malachite and epidote.

**Orthoclase**,  $\text{KAlSi}_3\text{O}_8$ . This monoclinic mineral twins according to several laws which may be stated as follows, in order of their importance:

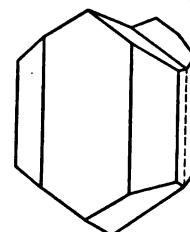


Fig. 201—Augite twinned on the orthopinacoid (100).

1. *Karlsbad law*. — twinning plane the orthopinacoid (100); composition face, the clinopinacoid (010), partially penetrating. This is an exceedingly common and characteristic form of twinning for this mineral (Figure 202 a and b).
2. *Bavero law*. — twinning plane and composition face, the clinodome (021) (Figure 202 d). Crystals of orthoclase showing this type of twinning are usually elongated in the direction of the clino axis.
3. *Mannebach law*. — twinning plane and composition face the basal pinacoid (001) (Figure 202 c).

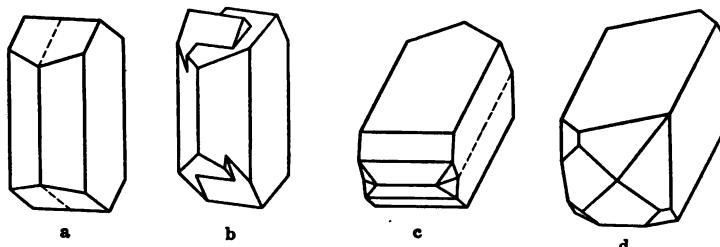


Fig. 202 — Orthoclase twinned by the three common laws for this species.

**Mica.** The minerals of the mica group twin according to the *mica law*, in which the twinning plane belongs to the zone of (110) and (001) and is at right angles to the basal pinacoid (001), while the composition face is (001). The individuals are placed on top of one another as shown in Figure 203. This rather complex law may seem to stand in conflict with the general statement according to which the twinning plane is always a possible crystal face. In this law the direction of the twinning plane can be stated in terms of possible crystal faces but the twinning plane itself is not a possible crystal face in the monoclinic system.

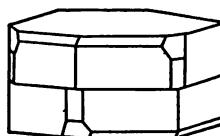


Fig. 203 — Mica contact twin by the mica law.

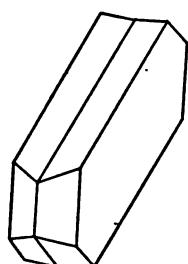


Fig. 204 — Simple twin according to the albite law — twinning plane the brachypinacoid (010).

This type of twinning can be readily observed on crystals of albite and also on the plagioclases generally. It gives rise to parallel striations on the best cleavage, or on the natural basal pinacoid. Figure 204 represents two indi-

### TRICLINIC SYSTEM

**Albite ( $\text{NaAlSi}_3\text{O}_8$ ).** The mineral albite is one of the feldspars and is known to twin according to the three laws previously mentioned for orthoclase but it most frequently twins according to the albite law which may be stated as follows:

Polysynthetic twinning with twinning plane and composition face the brachypinacoid (010).

This type of twinning can be readily observed on crystals of albite and also on the plagioclases generally. It gives rise to parallel striations on the best cleavage, or on the natural basal pinacoid. Figure 204 represents two indi-

viduals twinned in this way, while the complex polysynthetic aggregate is shown in Figure 189.

In the previous pages only a few of the commoner types of twinning have been referred to. Twinning according to many other laws has been observed and will be found on record as a part of the regular description of the various species.

**General Effect of Twinning.** It has already been observed that the effect of twinning is to increase the symmetry so that certain crystals of low symmetry as a result of complex twinning simulate a very high degree of symmetry. This is particularly well shown in the case of the monoclinic

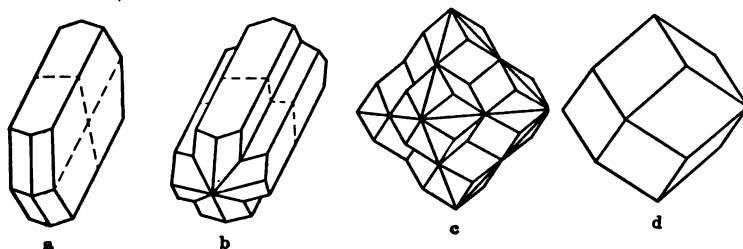


Fig. 205 — Mimicry of higher forms exhibited by the monoclinic mineral phillipsite as a result of twinning.

mineral phillipsite. Figure 205a represents a common form for phillipsite with twinning on the basal pinacoid (001). The two individuals interpenetrate so that the diagonally opposite quarters seen on the clinopinacoid belong to the same individual. If two such forms be still further twinned with the twinning plane the clinodome (011), then a complex form (Figure 205b) may be obtained from their interpenetration. The repetition of this process gives rise to forms represented by Figure 205c. It may be observed that the first type of twin simulates the symmetry for the rhombic system, the second the symmetry of the tetragonal system, and the last the symmetry of the cubic system. Sometimes on crystals of phillipsite the faces are so extended as a result of growth that complex forms similar to the rhombic dodecahedron are obtained.

Mallard was so impressed with the possibilities of produc-

ing forms of high symmetry by the combination of forms of lower symmetry as to suggest that all substances in their ultimate molecular structure were of the very lowest degree of symmetry and that the apparent higher symmetry is always pseudo-symmetry, being the result of twinning.

## CHAPTER XV

### IRREGULARITIES OF CRYSTAL SURFACES

In the previous pages crystals have been illustrated and described in their idealized form. Among crystals, as elsewhere, the ideal is very different from the real and is at best only approached by a relatively small proportion of the crystal individuals. It has already been pointed out that distorted crystals are exceedingly common. This is due to a variation in central distance among the faces composing the crystal form. The crystal drawings contained in the previous chapters are almost entirely of such a character as to represent only idealized crystals. There are other irregularities in regard to crystals which have not been acknowledged in our previous descriptions.

**Curved Faces.** Although crystals have been assumed to be always bounded by plane surfaces there are nevertheless many substances whose crystal faces are somewhat curved. This has been frequently observed on diamond (Figure 206),

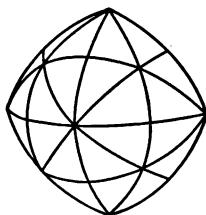


Fig. 206 — Curved surfaces exhibited by diamond.

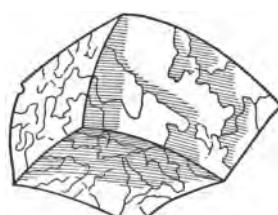


Fig. 207 — Curved crystal of siderite.

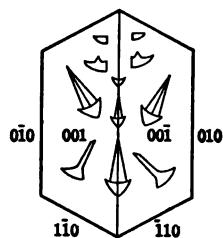
siderite (Figure 207), dolomite and calcite. The cause of the curvature of crystal faces has in some instances been explained as a result of solution or corrosion which has blunted the edges and corners by dissolving the substances from the edges and corners more rapidly than from the central parts

of the faces. This is the explanation advanced by Goldschmidt for the curvature commonly exhibited on crystals of diamond. The same explanation probably holds for apatite and calcite. In other instances the corners and edges of the crystal are sharper and more prominent than in the normal crystal. This can hardly be ascribed to corrosion of crystals once normal, since the general effect of corrosion is to reduce and to render less prominent the outstanding points and edges of the crystal. Forms such as that represented for siderite are apparently original forms assumed by the substances on crystallizing. The curvature may be due to the aggregation of sub-individuals in positions of approximate parallelism.

**Vicinal Planes.** In accordance with the law of rationality of axial parameters, only such faces occur on crystals as give rise to indices which are small whole numbers or zero. The examination of the inclination of the faces found upon crystals shows that it is usually possible to write the symbols of the various forms without employing indices larger than ten. Notwithstanding this generalization one frequently observes the presence of faces occupying a position *almost* parallel to certain faces of large development and simple index relationships. The occurrence on quartz of numerous faces almost parallel to the prism of the first order (1010) has been frequently observed. The following forms of this character have been noted: (12.0.12.1), (13.0.13.1), (15.0.15.1), (16.0.16.1), (17.0.17.1), (18.0.18.1) and (28.0.28.1). These forms are not more than four degrees from parallelism with the faces of the prism of the first order. Faces of this character are also well represented on many crystals of fluorite which appear at first sight to be completely enclosed by faces of the cube. On closer examination one frequently observes in addition to the cube faces very flat tetrahedra having the symbol approximately represented by (32.1.0). In some instances a large number of these peculiar planes may be found following one another and in this manner rounding edges and corners and giving rise to apparent curvature of crystal faces. In contrast to crystal faces whose indices are simple these complex planes are spoken of as vicinal planes.

**Striations.** Crystal faces are frequently striated by a series of parallel lines. A microscopic examination of such faces shows that these striations are due to ridges formed by the oscillation in the growth of the crystal. Cubes of pyrite are commonly marked by the presence of striae which are parallel to the cube edges. On adjacent faces these striae are at right angles to one another. (Figure 44). While certain portions of the faces of these crystals have undoubtedly the direction characteristic of cube planes the major portion of the surfaces consists of a pyritohedron. Striation on crystal surfaces is exceedingly common. On crystals of quartz the prismatic faces are usually striated at right angles to the vertical axis. In tourmaline many of the faces of the prismatic zone exhibit striations parallel to the vertical axis.

**Etching Figures.** When a crystal possessing smooth plane surfaces is subjected to the influence of a corrosive reagent



the solution takes place in a peculiar manner. First of all a few pits are formed which become larger as the corrosion is continued until finally their walls coming into contact with one another the pits disappear and small elevations are formed. These pits, which are called etching figures, can be easily produced

Fig. 208 — Etching figures on the plane surfaces of the crystals by using an appropriate corrosive in the laboratory. Figures 208 and 209 represent the form and symmetry of such pits on a basal pinacoid of albite and on certain faces of axinite respectively. The etching figures betray the physical symmetry of the face on which they occur. It has previously been shown

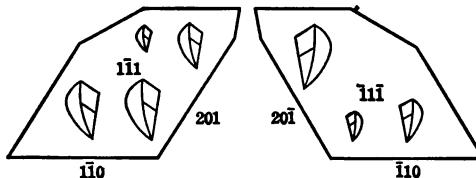


Fig. 209 — Etching figures on axinite showing the general absence of symmetry.

how the employment of this method may be utilized in the determination of crystal symmetry. Figure 210 represents

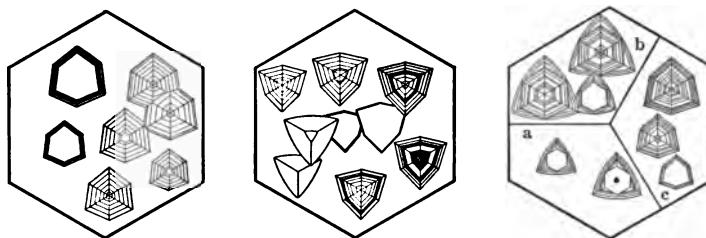


Fig. 210—Etching figures on the basal pinacoid of spangolite showing trigonal symmetry of this plane.—*After Penfield.*

the corrosion forms on the basal pinacoid of the mineral spangolite. Etching figures may be observed on the natural surfaces of many minerals, notably on quartz and topaz. It is assumed that these natural etching figures have been formed by some corrosive acting upon plane crystal surfaces.

## CHAPTER XVI

### CRYSTAL DRAWINGS

It is convenient to represent crystals by means of models or drawings. For certain purposes the model has the preference, while for compactness and for publication the drawing is essential.

Several different types of drawings are used by crystallographers. One of the commonest indicates the appearance of the crystal when it is viewed from a long distance and in the direction of some prominent series of zone edges. This type of drawing is known as a plan or orthographic projection. In practice the zone edge which represents the direction of vision for plans is parallel to the vertical axis  $c$ . Such a plan is bounded by a series of lines representing the edges

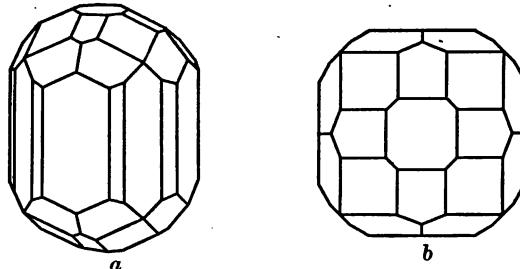


Fig. 211 — Clinographic and orthographic drawings of a crystal of wernerite.

between the faces composing the prismatic zone and the terminal faces. The edges between the various terminal faces are indicated by a network of lines within the boundaries set by the intersection of the terminal faces with those of the prismatic zone. The plan does not exactly represent the picture which could be obtained by a camera since the eye of the observer is represented as being at infinity — the lines which join the various corners shown in the plan to the point of vision being parallel to one another. It is not

possible to tell from an examination of such a plan whether the terminal faces of the crystal are flat or steep. This is one of the main disadvantages of representing crystals in this fashion. On the other hand when a crystal is terminated by a large number of faces, several of which lie almost in one plane, it is easier to distinguish the relationship of these faces to one another in the plan than in any other type of crystal drawing. Figure 211a represents a crystal of *wernerite* while Figure 211b is a plan parallel to the basal pinacoid. The relationship of the various terminal faces in this plan is more readily understood than in the oblique representation of the same crystal.

**Preparation of a Plan from a Gnomonic Projection.** In the gnomonic projection the intersection points of the face normals appear as points. Such a projection indicates to us neither the magnitude of the faces on the crystal nor the method of intersection of the faces to form edges. It is always possible, however, to learn from such a projection the direction of the edge between any pair of faces upon a crystal. The direction of a line representing the edge between a pair of faces in a plan parallel to the plane of the gnomonic pro-

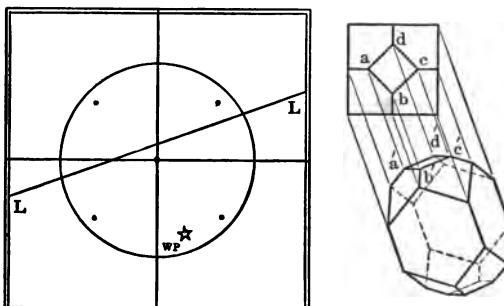


Fig. 212 — Sketch illustrating the method of preparing orthographic and clinographic projections of a crystal from the gnomonic projection.

jection is at right angles to the line connecting the projection points of the two faces. It is therefore easy to make a plan representing the faces which appear on a gnomonic projection. This may be conveniently carried out on part of the sheet containing the projection or upon another sheet of paper definitely fixed upon the drawing board with regard to the

sheet on which the projection appears (Figure 212). If the crystal represented in the projection be large it may be convenient to measure the length of the various edges and to make the drawing either in the natural scale or in some multiple of it. When the gnomonic projection is prepared from a crystal of small dimension it is necessary — by use of the telescope of the goniometer — to make a free-hand representation of the relative magnitude of the faces as they appear on the crystal. In this case the proportionate length of the edges between the faces contained in the plan are derived from the free-hand sketch. In the preparation, therefore, of such a plan it is necessary to have information on the following points:

1. The direction of the lines,
2. The length of the lines,
3. The position of the lines.

The direction is readily obtained from a gnomonic projection since the line representing the edge between any pair of faces is at right angles to the line connecting the projection points. The length of the edge line in the plan is dependent first upon its relative length on the crystal, and second, upon the scale in which it is desired to prepare the drawing. The scale may be the same as that shown upon the crystal — natural scale — or any multiple of the same. The position on the sheet of paper for the first line is arbitrarily selected. When this is represented in its proper position the lines which start from the ends of this first line are also fixed. From the ends of this first line others may be drawn and similarly the various lines required to complete the projection may be drawn from the ends of lines already fixed.

**Clinographic Drawings.** If a cube be viewed from a point in line with one of the axes of reference only one face is visible and it appears as a square (Figure 213a). The most information is obtained from drawings which represent the crystal when viewed from a point lying within one of the octants and not contained in a plane which bounds the octants. For many years crystallographers by general agreement have been accustomed to represent crystals by drawings

in which the direction of vision is the same for all systems. This direction may be best illustrated by reference to the cube. If instead of viewing the cube in the manner represented in Figure 213 a we revolve it about its vertical axis through a certain angle ( $18^{\circ} 26'$ ), we obtain a picture showing only two of the cube faces (Figure 213 b). Since it is evident that this is not an advantageous representation of the cube or any other crystal it has been agreed to elevate the eye  $9^{\circ} 28'$  so as to obtain a picture such as is represented in Figure 44. The picture shows three of the cube faces and represents the standard orientation for clinographic drawings.

A picture of this kind can be readily prepared starting from

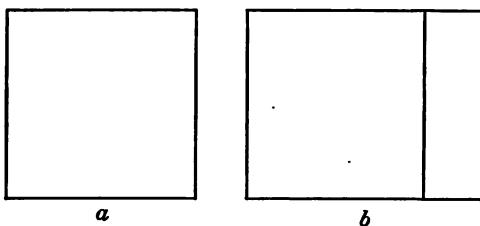


Fig. 213 — The cube as seen from different directions.

the gnomonic projection and plan. A line from the eye to the crystal centre would intersect the plane of gnomonic projection at a certain point W.P. which is marked by a star in Figure 212. A plane passing through the crystal centre and at right angles to this direction of vision intersects the plane of projection in the line L.L. All those faces whose projection points lie in front of the line L.L. will be directly visible, that is, on the side of the crystal toward the observer. According to Goldschmidt L.L. is known as the Leit Linie, while the point W.P. is the Winkel Punkt. It is evident that if the direction of vision be changed, that is, if a new Winkel Punkt be selected the Leit Linie will also change and certain faces may become visible in the picture while others disappear. Those faces whose projection points lie on the Leit Linie will be foreshortened in the picture to a single line, while for the other faces the nearer their projection points are to the line L.L. the more foreshortened do they appear. The point of vision is placed

at infinity so that all parallel edges are represented by parallel lines in the drawing. Having already prepared a gnomonic projection and a plan (Figure 212) as a preliminary we may proceed with the preparation of the clinographic projection. In the making of such a drawing we require to know the position, direction and length of the lines involved, which are fixed in the following manner:

1. To find the direction in a clinographic projection of the line representing an edge between two faces whose gnomonic projection points are indicated, draw a line through the projection points and produce, if necessary, to intersect the Leit Linie. The direction of the edge in question is at right angles to the line connecting this intersection point with the Winkel Punkt.
2. The corners represented in the plan are contained in the clinographic projection on lines from the corners at right angles to the Leit Linie. Through the point b draw a line normal to the Leit Linie, and arbitrarily select b' a point on this normal to represent the point b. Through the point c draw a normal to the Leit Linie and draw b'c' to represent the edge bc. Similarly c'd', d'a' and a'b' may be drawn, thus completing the outline in the clinographic projection for the basal pinacoid. The other lines involved in the projection may be drawn in the same way.
3. The lines representing the prismatic zone appear in the clinographic projection at right angles to the Leit Linie. The direction and position therefore of any one of these lines may be obtained by drawing a line at right angles from the point in the plan representing it.

## CHAPTER XVII

### ILLUSTRATION OF CRYSTALLOGRAPHIC INVESTIGATIONS

By permission of several crystallographic friends the extracts in this chapter have been selected from a number of publications indicating the problems with which crystallography has to deal and the most appropriate methods of solving them. It may be admitted that in previous chapters the information regarding the determinations of axial ratios and polar elements is not sufficient to make it possible for the student to conduct with confidence many of the investigations connected with the study of crystals. The extracts selected illustrate the method of procedure. Naturally the greatest difficulty is met in connection with the monoclinic and triclinic systems, where some, or all, of the axes of reference are oblique.

#### CUBIC SYSTEM

#### NEW FORMS OF SPERRYLITE

BY VICTOR GOLDSCHMIDT AND WILLIAM NICOL

(*American Journal of Science, June, 1903*)

The sperrylite examined by us during the summer of 1902 was obtained at the Vermillion Mine, Algoma District, Province of Ontario. This is the original locality from which was obtained the material examined by Wells and Penfield<sup>1</sup> and that described by Walker.<sup>2</sup> By "panning" the loose material at the mouth of the old shaft a heavy residue was obtained.

<sup>1</sup> This Journal (3), xxxvii, 67 et seq., 1889. *Zeitschr. f. Kryst.*, xv, 285 u. 290, 1889.

<sup>2</sup> This Journal (4), i, 110 et seq., 1896. *Zeitschr. f. Kryst.*, xxv, 561, 1895.

When this residue was examined by a needle and pocket lens the brilliant silver-white crystals of the mineral were readily separated from the magnetite, etc., occurring with them. The crystals are exceedingly small, but on account of their brilliant metallic lustre the measurement of the minute faces was easily possible. The crystals were measured on the two-circle reflecting goniometer, making use of the reduced signal.

Nine crystals were measured, which showed the following forms:

	Crystal System Regular					Pentagonal Hemihedrism									
Letter:	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Symb. Gdt.:	c	a	g	e	h	b	δ	k	m	q	B	p	u	ψ	x
Symb. Miller:	0	+ $\frac{1}{2}$ 0	- $\frac{1}{2}$ 0	+ $\frac{1}{2}$ 0	+ $\frac{1}{2}$ 0	- $\frac{1}{2}$ 0	-10	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	$\frac{1}{2}$	- $\frac{1}{2}$
	001	103	205	102	305	203	101	114	113	112	335	111	212	214	213

Of these forms, those designated with an asterisk are new for sperrylite. In addition to these, two other forms,

$$? z = + \frac{1}{2} \frac{1}{2} (315); ? D = + \frac{1}{2} \frac{1}{2} (326),$$

were observed, but not with absolute certainty.

Charles W. Dickson<sup>1</sup>, doing post-graduate work at the School of Mines, Columbia University, New York, has succeeded in separating sperrylite crystals from the unaltered chalcopyrite from the Victoria Mine, in the Sudbury District. He reports by letter, "I found, on examination of two of the best crystals, the trapezohedrons 211 and 411, besides the originally described forms."

The form e occurs on all the crystals measured, having only half the number of faces developed, in accordance with pentagonal hemihedrism. This form has been regarded as *positive*, and the *positive* or *negative* character of the other forms has been determined in accordance with this. This criterion may be accepted as decisive. On account of the minute size of the crystals it was not possible to attempt to decide this point by etch-figures.

The nine crystals measured showed the following combinations:

#### Combinations

Crystal No. 1: c . g . e . b . . . m q . p u . . . . Fig. 2.  
 Crystal No. 2: c . . . e . . . . . . p . . . . .  
 Crystal No. 3: c . . . e . . . . . q B p . . . . .

<sup>1</sup> This Journal (4), xv, 137, 1903.

Crystal No. 4: c . . e . b' δ . m q B p u ψ x . . Fig. 3.

Crystal No. 5: c . . e . . δ k . q . p u ψ . (z). Fig. 4.

Crystal No. 6: c . . e h . . . . . . . . . . Fig. 5.

Crystal No. 7: c . . e . . δ k m q . p . . . . (D)

Crystal No. 8: c a . e . . . . . . . . p . . . . .

Crystal No. 9: c . . e . . . . . q . p . . . . .

From this table of combinations the relative frequency and importance of the individual forms can be observed.

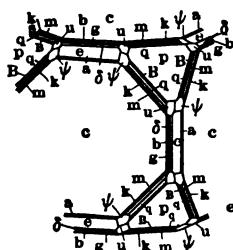


Fig. 1

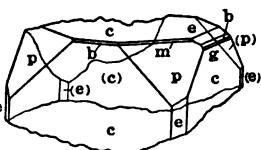


Fig. 2

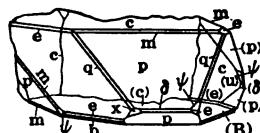


Fig. 3

The following arrangement shows the relative importance:  
**c** always present, usually predominating.

**p** almost always present, usually smaller than **c**, sometimes of equal importance with **c**, now and then predominating.

**e** always present, but less important by far than **p**.

**q** **δ** **u** **m** very small, but occurring not seldom.

**a** **g** **h** **b'** **k** **B** **x** occurring seldom and unimportant.

**z** **D** scarce, faint and uncertain.

Up to the present time, from the measurements of Penfield and Walker, the following forms were known:

**c** = 0 (001); **δ** = 10 (101); **e** = +  $\frac{1}{2}$  0 (102); **p** = 1 (111).

These are, in fact, the most important forms. In addition Walker gives the form  $\frac{1}{2}\bar{1}$  (5.2.10), determined from measurements of plane angles. This form was not found on any of the nine crystals measured. It lies in the zone  $+\frac{1}{2}q$ , between **e** =  $\frac{1}{2}0$  and **q** =  $\frac{1}{2}$ , — an important zone for this mineral. It may perhaps be identified with the form **ψ** =  $\frac{1}{2}\bar{4}$ . In the same zone lies the uncertain form **D** =  $\frac{1}{2}\bar{3}$  and the well defined form **u** =  $\frac{1}{2}1$ . Figures 2, 3, 4 and 5 show crystals Nos. 1, 4, 5 and 6. Figure 1 gives an ideal picture of all the forms. Owing to the difficulty of drawing so many

narrow faces and reproducing them by lithographic plate, only part of the crystal is shown. The forms x and h are wanting, as their existence was decided with certainty by measurements made after the drawing was prepared. In Figures 2-5 the attempt has been made to preserve the proportions of the faces as they actually occur in nature.

The measurement, or rather *the discussion of the results of the measurement*, offered several difficulties which can be referred to the small size of the crystals and especially of the subordinate faces. These faces are not only absolutely but also relatively small, that is, when compared as to size, with the principal faces, c and p, they stand far in the background. This may be readily seen from an inspection of the figures. In discussing the measurements in the gnomonic projection,

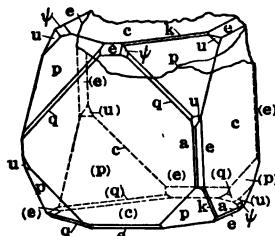


Fig. 4

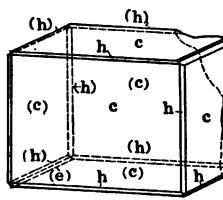


Fig. 5

the following unexpected phenomenon was observed. In cases where the principal faces c and p occupied their exact position, showing that the crystal had not been disturbed in its growth, yet the signal reflections from the subordinate faces were displaced in many ways, and this, too, when these faces were well defined as to lustre and perimeter. The minuteness of the subordinate faces is exceptional. It is interesting to note what minute faces can be measured by the excellent instruments of the present day. The following measurements were obtained under the microscope:

Crystal No. 4, Fig. 3.

The large upper c face,	longest diameter = 0.8 mm
The large front p face,	" " = 0.6
The upper e face, to the right from p,	" " = 0.06
The lower $\psi$ face, to the right from p,	" " = 0.03
The lower x face, to the left from p,	" " = 0.03
The lower $\delta$ face,	length = 0.3 mm; width = 0.015

The displacements and consequent variations in the case of such small faces may have two causes:

1. Optical displacement and diffusion of the signal reflection owing to bending of the light. The smaller the face the stronger is this effect and the less the certainty in determining the position of the face.

2. Curvature of faces near the outlines. This is a consideration of importance in connection with the origin of the crystal and demands special study. Sperrylite itself may perhaps furnish suitable material for this on account of its perfect metallic lustre, which makes it possible to secure good signal reflections from even minute faces: on account of the simplicity of the crystal system and the certainty of the elements and calculated angles: on account of the perfect formation of the principal faces, the great variety of kinds of faces and the large number of individual faces, which are incident to the regular system. It may be said: *the degree of perfection of a face is a function of its absolute and relative size.* Very large faces are usually imperfect since their parts and the parts of the layers underneath have a development history exposed to different influences. The faces of smaller crystals are, as a rule, better formed than those of larger crystals. Below a certain size the faces again become more imperfect. This may be assigned to two causes:

1. The relatively small faces are those derived from higher complication. They are weak and more easily displaced. They are mobile, like the leaves and branches in the wind, while the trunk is unmoved.

2. In the case of small faces the marginal parts are relatively large. The marginal parts, however (on edges and corners), are the places at which the particles are bounded only on one side by securely situated particles which are, as well, oriented themselves and have an orienting influence on their neighbors. This is the place of the manifold formation of varying resultants by complication.<sup>1</sup> The midfield of not too great a face is therefore usually the smoothest. Convexity readily occurs at the perimeter and corners. Small

<sup>1</sup> Compare Zeitschr. f. Kryst., xxix, 47, 1897.

faces lying close together and very little inclined towards one another frequently shade into each other by rounding. It may happen that *only* the border part is present, no smooth middle part has been formed. If, now, the border part is large relatively, it furnishes much light for the signal reflection. In this way the signal reflection from the inner portion of the face, when this is small and of low brilliancy, may become darkened and eclipsed by that of the border parts. If the faces are too small it is not possible to shut off the border portions by means of the shutter at the eye-piece of the telescope. It is not advisable at this place to go more deeply into these interesting points. They should form the ground of a special investigation.

The discussion of the position of the faces and the symbols of the subordinate ones must be undertaken with caution on the above mentioned grounds. Considerable experience is necessary in order to be able to separate here the certain from the uncertain. One must also, by measuring numerous crystals, accustom one's self to the peculiarities of the kind of crystal at hand. On the one hand, there is the temptation to symbolize the positions of the diffused signal reflections. This may lead to great confusion. On the other hand, there is the danger of demanding too great precision, and therefore of finding here only the principal forms c, p, e. It was attempted by careful discussion of the measurements and their picture in the gnomonic projection to obtain as many certain and typical forms as possible. The forms here given as certain may be accepted. Those indicated by (?) viz., z and D, possess a high degree of probability. Under these circumstances a greater variation than usual must be allowed between the positions of the faces as determined by measurement and by calculation.

## HEXAGONAL SYSTEM

## APATITE FROM MINOT, MAINE

BY JOHN E. WOLFF AND CHARLES PALACHE

*(Proc. Am. Academy of Arts and Sciences, March, 1902)*

In the summer of 1901, while prospecting for tourmaline or other gem minerals on the farm of Mr. P. P. Pulsifer in Minot, Maine, a pocket was opened in the granite containing the material here described. It was first brought to our notice by Mr. C. L. Whittle, formerly of this Department, and the whole was subsequently acquired by the Harvard Mineralogical Museum.

This find is noteworthy for the unusually rich purple color of the crystals, and the purity, crystalline perfection, and abundance of the material, which comprises about two thousand loose crystals or fragments of crystals with a total weight of over a kilogramme, and about a dozen large groups of crystals on the matrix. Of the loose crystals about three hundred show at least one perfect termination, five hundred are slightly less perfect, and the rest imperfect or fragmentary.

CRYSTALLOGRAPHY<sup>1</sup>

The apatite crystals are in general of pronounced prismatic habit, the average size being about 1 cm. in height and 0.5 cm. in diameter. Crystals larger than this are, however, common, the largest measuring nearly 3 cm. in height and diameter. Crystals smaller than the average, which are also numerous, tend to assume a more or less rounded habit by nearly equal development of prismatic and terminal planes.

The crystals are generally so implanted upon a terminal face that one end has developed freely, and the fact that over three hundred loose crystals with complete single termination and prism zone were obtained from the collection shows how prevailing is this habit of growth. Occasionally the attachment to the matrix is by a prism plane, and then both terminations are developed.

<sup>1</sup> By C. Palache.

The forms observed were as follows, the letters used being those of Dana:

c (0001), m (1010), a (1120), h (2130), z (3031), y (2021), x (1011), r (1012), w (7073), s (1121),  $\mu$  (2131),  $\mu_1$  (3121).

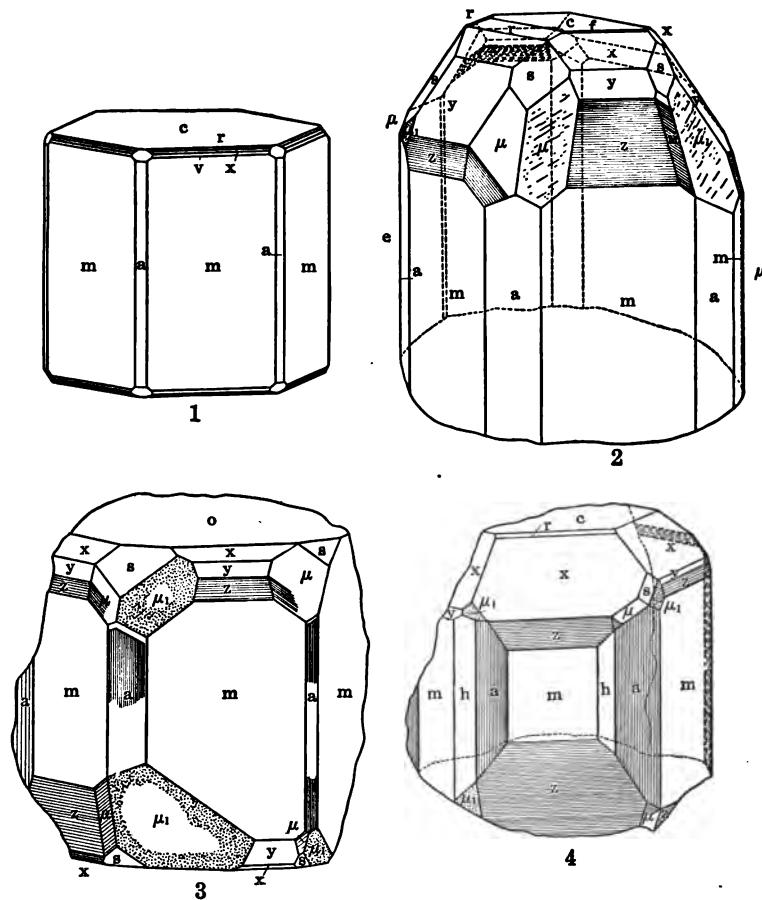
Four crystals were carefully measured on the two-circle goniometer and the same forms found on all. The results of measurement of the better developed forms agreed so well among themselves that it seemed worth while to calculate the axial ratio from the better readings, and this was done, using the forms y, x, r and s. The following table shows the average angle to the base from each of these, the ratio calculated for each crystal and the average ratio obtained:

	Angle from 0001 to 2021.	No. of Faces	d. <sup>1</sup>		Angle from 0001 to 1021.	No. of Faces	d. <sup>1</sup>
Cryst. 1 . .	59°29'	5	3'	Cryst. 1 . .	23°00'	6	1'
" 2 . .	59°29½'	5	2'	" 2 . .	23°00'	5	5'
" 3 . .	59°30½'	6	3'	" 3 . .	23°00'	3	0'
" 4 . .	59°28½'	5	3'	" 4 . .	22°59½'	4	3'
Angle from 0001 to 1011.				Angle from 0001 to 1121.			
Cryst. 1 . .	40°18'	6	3'	Cryst. 1 . .	55°45'	6	1'
" 2 . .	40°19'	5	5'	" 2 . .	55°46'	6	3'
" 3 . .	40°19'	5	6'	" 3 . .	55°46'	4	3'
" 4 . .	40°18'	5	2'	" 4 . .	55°45'	6	4'
Crystal 1, from 23 measurements, $p_0 = 0.848307$							
Crystal 2, from 21 measurements, $p_0 = 0.848739$							
Crystal 3, from 18 measurements, $p_0 = 0.848753$							
Crystal 4, from 20 measurements, $p_0 = 0.848148$							
Average from 82 measurements, $p_0 = 0.848476$ or a:c = 1:0.734800							
Angle calculated from $p_0 = 0.848476$ , 0001 to 2021 59° 29' 22"							
0001 to 1011 40° 18' 50"							
0001 to 1012 22° 59' 19"							
0001 to 1121 55° 45' 59"							

Two types of combinations may be distinguished among these crystals. One of these is represented in Figure 1, and consists essentially of the prism of the first order and the base, the edges modified by narrow planes of the forms a, s, r, x and y. Crystals of this type are not uncommon and often

<sup>1</sup> d is the difference in minutes between largest and smallest readings for faces of any form.

show double terminations. They merge, however, by slight gradations into the second type, more characteristic for the locality, shown in Figures 2 and 3. Here the pyramidal planes become more prominent and the most notable feature is the simultaneous occurrence of the right and left third order pyramids, giving the appearance of the normal dihexagonal pyramid.



Figs. 1 to 4

The different forms may be characterized as follows:  
 c (0001) always present, generally large, brilliant, and plane giving perfect reflections.

$m$  (1010) always present, generally dominant, brilliant, and generally plane but sometimes faintly striated vertically.

$a$  (1120) generally present but narrow and commonly dull from deep striation, the striæ vertical and bounded by faces of adjoining planes of  $m$ . Occasionally the striations stop abruptly in the centre or near the boundaries of a face as shown in Figure 3, or they may be wholly lacking, in which case the face is brilliant and gives good reflections.

$h$  (2130) rarely developed and then narrow as shown in Figure 4. Surface plane, not involved in striations on  $a$ .

$r$  (1012),  $x$  (1011) and  $y$  (2021) all nearly always present with all their faces, in varying proportions and often large, faces always brilliant and free from striations, giving perfect reflections.

$w$  (7073) observed but once as a line face in the zone between  $y$  and  $m$ .

$z$  (3031) generally present only as a deeply striated face, sometimes very large as in Figure 4, giving no reflection but determined by its zonal relation to  $\mu$  and  $\mu_1$ . The striæ bounded by faces parallel to adjoining planes of  $m$  and  $y$ . Narrow faces of  $z$  giving faint reflections sometimes present on the edges of the striæ nearest to  $m$ .

$s$  (1121) always present with brilliant faces, often large.

$\mu$  (2131) and  $\mu_1$  (3121) are both present on many crystals, but vary widely in size, quality and regularity of development. Generally the faces of both are dull and the forms are then indistinguishable. On some crystals their faces are brilliant and reflecting but grooved or pitted, and a constant difference in the character of these markings was found by which, when they were not too far developed, the two forms could be distinguished. On  $\mu$  the markings ordinarily take the form of sharp grooves parallel to the intersection of  $m$  and  $\mu$  as shown in Figures 2 and 3. The grooves seem to be in a way continuations of the striæ on the faces of  $z$ , for they never extend beyond the intersection of  $\mu$  with that face, and are absent if  $z$  is not developed. The grooves are bounded by faces parallel to adjacent planes of  $s$  and of  $m$ . Very often they stop short in the middle of the face as shown in Figure 3.

On  $\mu_1$  the markings are in the form of irregular pits or curv-

ing grooves, sometimes showing approximate parallelism to the intersection of  $m$  and  $\mu_1$  but with an irregularity giving them a character wholly different from the lines on  $\mu$ . No constant difference could be observed in the brilliancy of the reflecting portions of faces of the two forms, nor in their relative size. Both are irregular in their occurrence on individual crystals, lacking nearly always some of their faces. As shown in the figures, both may present on the same crystal faces of very unequal size which in some cases are so large as to dominate the termination of the crystal.

The occurrence of third order pyramids in apparently holohedral combination has been observed on apatite from various localities, notably Knappenwand, Tyrol,<sup>1</sup> Ala, Piedmont<sup>2</sup> and Elba.<sup>3</sup> But in none of the crystals described does there appear to have been any observable difference between the faces of the right and left forms by which they could be distinguished.

Reference has been made in the preceding pages to striations which appear quite constantly on certain faces of the apatite. They are a striking feature of the crystals and the attempt has been made to reproduce them in the drawings. Their most pronounced development was on the largest crystal of the collection, which is reproduced in Figure 4; the striations on the faces of  $z$  and of  $a$  were almost equally strong and gave the crystal a curiously tetragonal aspect when inspected casually. On both of these forms the striations are doubtless growth forms, the result of oscillatory combination, on  $a$  of adjacent faces of  $m$ , and on  $z$  of planes of  $m$  and  $y$ . The markings on the faces of  $\mu$  and  $\mu_1$  seem to have a different character, however. The irregularity of their development, appearing on some faces as mere grooves or pits, on others invading the whole face and reducing it to a dull surface, indicates that they are rather the result of etching by some agent which has attacked the crystals after they were formed.

<sup>1</sup> C. Klein, Neues Jahrb. Miner., 1871, 485; 1872, 121.

<sup>2</sup> G. Strüver, Att. Acc. Torino, 3, 125, 1867; 6, 363, 1871; Rendic. R. Acc. Lincei, Roma, 1899, 8 (1), 427-434.

<sup>3</sup> E. Artini, Rendic. R. Acc. Lincei, Roma, 1895, 4 (2), 259.

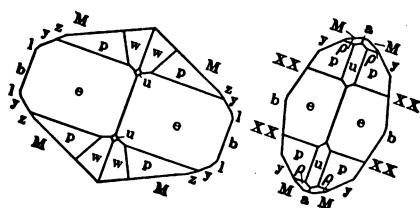
## RHOMBIC SYSTEM

NOTES ON GOETHITE<sup>1</sup> (Abstract)

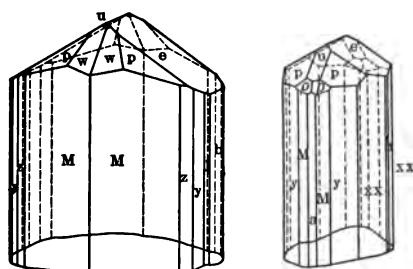
BY V. GOLDSCHMIDT (Heidelberg) AND A. L. PARSONS (Toronto)

(American Journal of Science, March, 1910)

In the summer of 1908 Mr. Parsons collected some specimens of goethite which occurs in veins in carboniferous shale at Walton, N.S. The veins are brecciated and the centre filled with calcite. When the calcite was dissolved in hydrochloric acid the goethite was left as a druse of bright crystals. Three crystals were measured. The observed forms are :



$b = 0\infty (010);$   
 $M = 2\infty (210);$   
 $a = \infty 0 (100);$   
 $y = \infty (110);$   
 $XX = \infty 4 (140);$   
 $e = 01 (011);$   
 $u = 10 (101);$   
 $p = 1 (111);$   
 $\rho = 31 (311).$



Figs. 1 and 2

The form designated as XX appears in all the crystals as the face with the greatest development. This face is striated lengthwise and is not a single face but a transition face or "Scheinflaeche," and it gives a bright band of light in the prism zone with the

angle  $\phi$  varying from  $3^\circ$  to  $20^\circ$ . In this band are bright points which imperfectly indicate the positions of the faces  $\infty 3$  (130),  $\infty 4$  (140),  $\infty 5$  (150),  $\infty 6$  (160),  $\infty 8$  (180),  $\infty 12$  (1.12.0). The harmonic discussion of the characteristic indices in this series gives  $\infty 3$ ,  $\infty 4$ ,  $\infty 6$ , and  $\infty 8$  as the points of best position, but no characteristic letter is assigned to them until

<sup>1</sup> Abstract of a paper in Groth's Zeitschrift für Krystallographie.

they have been determined by single distinct reflecting faces. The symbol XX is used for the series and in Figure 2,  $\infty 4$  represents the series.

For comparative purposes three crystals from Lostwithiel, Cornwall, were measured and the habit and faces are shown in Figure 1. The observed forms are  $b = 0\infty (010)$ ;  $M = 2\infty (210)$ ;  $x = \frac{1}{3}\infty (430)$ ;  $y = \infty (110)$ ;  $l = \infty 2 (120)$ ;  $u = 10 (101)$ ;  $e = 01 (011)$ ;  $p = 1 (111)$ ;  $w = \frac{1}{3}\infty (413)$ . All the forms except x were present on every crystal. The new form  $x = \frac{1}{3}\infty (430)$  was present on two crystals and for the best reflections gave angle  $\phi 55^\circ 6'$  and  $55^\circ 33'$ . A poor reflection gave one face as  $57^\circ 27'$ . Angle  $\rho$  was in every case  $90^\circ$ . The calculated angles are  $\phi 55^\circ 26'$ ,  $\rho = 90^\circ$ . The face is small but well defined, and may be regarded as well established. The form  $w = \frac{1}{3}\infty (413)$  is also new and is present on all three crystals with four faces on each. The faces reflect well and give a distinct signal cross, but it is worthy of remark that the reflection is slightly yellow while the others are white. The angles agree well among themselves, but in every case the angle  $\rho$  is  $20'$  to  $69'$  less than the calculated angle. The calculated angles are  $\phi = 77^\circ 4'$ ,  $\rho = 42^\circ 6'$ . Considering the good character of the reflection and the sharpness of the faces, this difference is not easily understood, but the form may be considered as well established.

Heidelberg, Aug. 14, 1909.

#### MONOCLINIC SYSTEM

#### COLEMANITE FROM SOUTHERN CALIFORNIA

A DESCRIPTION OF THE CRYSTALS AND OF THE METHOD OF MEASUREMENT WITH THE TWO-CIRCLE GONIOMETER

BY ARTHUR S. EAKLE

(*Bulletin of the Department of Geology, University of California, 1902*)

#### DESCRIPTION OF CRYSTALS

*Introduction.* — The mineralogical collection of the University of California includes several excellent specimens of colemanite, showing beautiful crystals in the druses and geodes of the massive material, and the abundance of good

measurable crystals afforded an opportunity to make a very complete crystallographic study of this borate. The suite of crystals selected for measurement included probably all of the habits, and possibly all of the forms, possessed by the mineral.

The measurements were made with the two-circle (*Zweikreisige*) goniometer, designed by Goldschmidt. Since the method of measurement with this instrument and the gnomonic projection are as yet not generally understood by American readers, a detailed statement of the work follows the description of the crystals, which will make clear the steps used in the calculation and projection of the forms.

Colemanite was first discovered in Death Valley, Inyo County, California, in 1882 and in the following year the more extensive deposits were found in the Calico District, about five miles from Daggett, San Bernardino County. Most of the fine geodal specimens are from the Calico District and it is presumed that the crystals described here are from this locality, as the labels indicate.

The crystal forms of colemanite were first described by Jackson.<sup>1</sup> His description of the crystals was quite complete and forms the basis of what is at present known regarding their forms. Some of his crystals were from the Death Valley deposits, but the majority came from the Calico District.

Others who have measured the crystals are: Hiortdahl,<sup>2</sup> who published a short description of the forms and optical properties, with a chemical analysis of a few crystals from Death Valley; Bodewig and vom Rath,<sup>3</sup> who likewise described some of the Death Valley crystals, and also gave an account of the origin of the deposits; and Arzruni,<sup>4</sup> who described one crystal.

The crystals commonly line geodal-shaped cavities and some are quite large, one in the collection having a width of ten centimetres. They are colorless transparent, to white,

<sup>1</sup> A. W. Jackson. Bull. of Cal. Acad. Sciences, 1885, No. 2, 2-36, and 1886, No. 4, 358-365.

<sup>2</sup> Th. Hiortdahl. Zeitschrift für Krystallographie, 1885. 10, 25-31.

<sup>3</sup> C. Bodewig und G. vom Rath. Idem, 179-186.

<sup>4</sup> A. Arzruni. Idem, 272-276.

although a few are stained yellowish by iron. Most of them are attached to the matrix by one end of the vertical axis, leaving one end well terminated by front and rear forms. This circumstance rendered but one mounting on the goniometer usually necessary in order to measure all of the forms on a crystal, and measurements consequently could be rapidly made. Complete measurements were made of thirty crystals, and many more were examined for additional forms.

*Elements.* — The axial ratio and angle  $\beta$  generally accepted for colemanite are those determined by Jackson. He derived his elements from the measurements of one crystal, but subsequently verified his calculations by the examination of more crystals. By the two-circle method of measurement all of the readings can enter into the computation of the axial lengths; consequently a ratio obtained by using a large number of readings from the best faces is presumably more exact than when calculated from a few interfacial angles. As shown later in the detailed description of the work, the axial ratio and angle  $\beta$  obtained for colemanite by the writer were as follows:

$$a:b:c = 0.7768:1:0.5430; \beta = 110^\circ 7'.$$

For comparison the elements calculated by others are as follows:

Jackson,  $a:b:c = 0.7755:1:0.5415; \beta = 110^\circ 13'$ .

Hiortdahl,  $a:b:c = 0.7747:1:0.5418; \beta = 110^\circ 13'$ .

Bodewig and vom Rath,  $a:b:c = 0.7759:1:0.5416; \beta = 110^\circ 16\frac{2}{3}'$ .

*Forms.* — The number of forms observed was forty-seven, of which thirteen were new. The forms are arranged in three columns below, those in the last column being new. The lettering is the same as that given by Dana in his "System of Mineralogy." Goldschmidt's and Miller's symbols are given for each form.

The commonest forms are c, b, a, m, t,  $\alpha$ ,  $\kappa$ , h,  $\beta$ , y, v, d, and o. Less common, yet of quite frequent occurrence, are k,  $\lambda$ , i,  $\sigma$ ,  $\omega$ ,  $\epsilon$ ,  $\psi$ , x, U. The remainder of the known forms are rare: Q occurred four times,  $\gamma$  three times, r, h, V, twice, and e, q, B,  $\Theta$ , each once. W is common on one type of the crystals, but occurred only twice in the others.

Letter	Symbol.		Letter	Symbol.		Letter	Symbol.	
	Gdt.	Miller.		Gdt.	Miller.		Gdt.	Miller.
<i>c</i>	0	001	<i>k</i>	+ 31	311	<i>l</i>	3 $\infty$	310
<i>b</i>	0 $\infty$	010	<i>C</i>	+ 10.1	10.1.1	<i>p</i>	+ 30	301
<i>a</i>	$\infty$ 0	100	<i>e</i>	+ 12	121	<i>g</i>	- 40	502
<i>t</i>	2 $\infty$	210	<i>s</i>	+ 3	331	<i>f</i>	- 80	801
<i>m</i>	$\infty$	110	<i>ω</i>	+ 13	131	<i>φ</i>	+ $\frac{1}{2}$ 1	522
<i>z</i>	$\infty$ 2	120	<i>y</i>	- 1	111	<i>p</i>	+ $\frac{1}{2}$ 2	142
<i>H</i>	$\infty$ 3	130	<i>v</i>	- 2	221	<i>n</i>	+ 14	141
<i>κ</i>	01	011	<i>q</i>	- 3	331	<i>u</i>	+ $\frac{1}{2}$ 1	164
<i>a</i>	02	021	<i>B</i>	- 41	411	<i>μ</i>	+ $\frac{1}{2}$ 5	165
<i>λ</i>	+ 20	201	<i>Θ</i>	- 31	311	<i>η</i>	+ 1 $\frac{1}{2}$	232
<i>V</i>	+ 10	101	<i>o</i>	- 21	211	<i>P</i>	- $\frac{1}{2}$ 1	123
<i>i</i>	- 10	101	<i>γ</i>	- 32	321	<i>w</i>	- $\frac{1}{2}$ 4	182
<i>h</i>	- 20	201	<i>ε</i>	- 23	231	<i>s</i>	- 34	341
<i>W</i>	- 30	301	<i>d</i>	- 12	121			
<i>ψ</i>	- 40	401	<i>Q</i>	- 24	241			
<i>U</i>	- 60	601	<i>x</i>	- 13	131			
<i>β</i>	+ 1	111	<i>r</i>	- 1 $\frac{1}{2}$	232			

The unit prism *m* is the predominating form, and the other prismatic forms are very narrow. Usually the rear face of *a* is much broader than the front one; *b* is usually present as cleavage faces, and when the natural face does occur it is small. When the base *c* and orthodome *h* are broad, the remainder of the terminal forms are small and usually few in number; on the other hand, when these two forms are absent, or very narrow, the other terminal forms are well-developed. As a rule the negative forms, that is, the upper rear forms,<sup>1</sup> are larger and better developed than the positive ones. A steep pyramid occurs on a few of the crystals, but the faces gave very poor reflections. The best readings indicate the symbols {10.1.1} for the form, which are the same as determined by Jackson and cited as doubtful.

Eight forms given by Jackson were not observed by the writer. These are {370}, {10.19.0}, {19.19.6}, {771}, {412}, {731}, {721} and {711}. According to him, the faces of

<sup>1</sup>This usage of the terms positive and negative as applied to the monoclinic system is different from that generally used in this book. T. L. W.

all these forms were extremely narrow, and gave either broad bands of light or no wedge-reflections whatever. In all probability  $\{370\}$  and  $\{10.19.0\}$  are  $\{120\}$ , and  $\{19.19.6\}$  is  $\{331\}$ . Many of his original crystals were examined by the writer for these forms, but they could not be identified.

*New Forms.* — With the exception of  $\{341\}$ , each of the new forms was observed but once.

$l = 3\infty\{310\}$ . This form was represented by a narrow face between  $\{100\}$  and  $\{210\}$ . The reflection was bright and good.

	$\phi$	$\rho$
Measured	76°29'	90°0'
Calculated	76 20	90 0

$\rho = +30\{301\}$ . This was a narrow bright face, but the image was only fair, and the  $\rho$ -angle varied somewhat from the calculated one.

	$\phi$	$\rho$
Measured	90°2'	69°33'
Calculated	90 0	68 57

$g = -\frac{5}{2}0\{\bar{5}02\}$ . This form occurred as a small triangular face, giving a good reflection.

	$\phi$	$\rho$
Measured	90°7'	56°20'
Calculated	90 0	56 12

$j = -80\{\bar{8}01\}$ . This was a very narrow dome face, and the reflection was poor. The face occurred between  $\{\bar{1}00\}$  and  $\{\bar{2}01\}$  in the same zone.

	$\phi$	$\rho$
Measured	89°23'	80°13'
Calculated	90 0	79 51

$\phi = +\frac{5}{2}1\{522\}$ . This form occurred as a narrow face between  $\{311\}$  and  $\{111\}$ . It gave a fair reflection, but the angles vary somewhat from those calculated for the symbols. The symbols are the simplest and probably the most correct for the form.

	$\phi$	$\rho$
Measured	75°49'	65°46'
Calculated	76 18	66 26

$p = + \frac{1}{2}\{142\}$ . This was a very small triangular face with a bright image.

	$\phi$	$\rho$
Measured	34°33'	52°47'
Calculated	34 12	52 43

$u = + \frac{1}{2}\{164\}$ . This occurred on the same crystal as the preceding, and was also small and bright.

	$\phi$	$\rho$
Measured	34°23'	44°28'
Calculated	34 8	44 32

The next five of the following forms occurred on the same crystal.

$n = + 14\{141\}$ . Two faces of this form occurred. They were both well developed and gave good reflections.

	$\phi$	$\rho$
Measured	27°11'	67°53'
	27 9	67 33
Calculated	27 5	67 42

$\mu = + \frac{1}{2}\{165\}$ . This was a very bright small face which gave a good reflection.

	$\phi$	$\rho$
Measured	38°11'	39°40'
Calculated	38 19	39 43

$\eta = + \frac{1}{2}\{232\}$ . This was a narrow face with a bright reflection.

	$\phi$	$\rho$
Measured	53°41'	54°10'
Calculated	53 44	54 1

$P = - \frac{1}{2}\{\bar{1}23\}$ . This form was represented by a small, well-developed face which gave a bright image.

	$\phi$	$\rho$
Measured	18°30'	21°10'
Calculated	18 5	20 51

$s = - 34\{\bar{3}41\}$ . Two large well-developed faces of this form occurred on this crystal, and it was afterwards also observed on one other. The reflections were good.

	$\phi$	$\rho$
Measured	1 { 40°50'	70°50'
	40 48	70 46
Calculated	2 { 40 32	70 52
	40 40	70 45

$w = -\frac{1}{2}4\{\bar{1}82\}$ . The edge between  $\{1\bar{3}1\}$  and  $\{110\}$  was replaced by a narrow face which was somewhat rounded, and did not lie exactly in the same zone with these two forms. The symbols are the nearest simple ones to agree with the readings, although the measured and calculated angles do not closely agree.

	$\phi$	$\rho$
Measured	0°16'	63°41'
Calculated	0 09	64 16

*Crystal Habit.* — Four quite distinct habits are noticeable, and these will be designated as Habits 1, 2, 3 and 4.

Habit 1. — Crystals of this habit are characterized by a broad base (001) and rear orthodome ( $\bar{2}01$ ), the two faces meeting in a long edge, almost the width of the crystals. The prism (110) is long and almost meets the base, the unit pyramid (111) being merely a narrow truncating form. The pyramidal forms are small and grouped at the extreme right and left corners of the crystal. The clinodomes (011) and (021) are often absent. This habit exhibits the monoclinic symmetry of the crystals in a more pronounced way than the other habits do. This type of crystal is seen in Figure 1, Plate 2. The rear orthopinacoidal face ( $\bar{1}00$ ) is generally broad like the dome ( $\bar{2}01$ ). The clinopinacoid occurs only as broad cleavage faces, and many of the crystals are cleaved into tabular plates parallel to this form.

Habit 2. — In this habit the base and orthodomes are either very narrow or completely wanting. The clinodomes and rear pyramids are large and about equal in size. The crystals have a characteristic pointed appearance at the ends of the orthodiagonals, the points being truncated by small natural faces of the clinopinacoid. Most of the crystals seem to be of this habit. Figure 2, Plate 2, shows this habit.

Habit 3. — In this habit the front upper terminal faces are

quite small, while the unit prism faces and the orthodome (201) are large. This gives a somewhat flattened appearance to the crystals, and at the same time causes them to be pointed at the ends of the vertical axis. This habit is quite striking, and the specimens are quite suggestive of dog-tooth spar. This type is seen in Figure 3, Plate 2.

Habit 4. — The crystals of this habit consist essentially of the prism (110) and a broad steep orthodome (301). The combination of these two forms makes a very thin wedge-shaped crystal with sharp edges. Some of the other forms also occur, but they are quite subordinate to these, and do not cause a variation in the habit. The dome (301) has a rounded or wavy surface and seems to grade into a still steeper dome, probably (401) or (601). The form (301) is characteristic of this habit, and was only observed twice on crystals of the other habits. Figure 4, Plate 2, shows this habit with curved edges resembling spear heads. This habit was first described by H. S. Washington.<sup>1</sup>

*Combinations.* — The crystals are so rich in forms that many of them have thirty or more faces. The various combinations observed were as follows:

1.  $c, a, t, m, \kappa, a, h, W, \psi, \sigma, \beta, y, v, d, C, o, f.$
2.  $b, a, t, m, z, \kappa, a, \sigma, \beta, v, d, p, u.$
3.  $c, b, a, t, m, \kappa, a, h, \sigma, \beta, y, v, d, x, o, \epsilon, w, Q.$
4.  $a, t, m, h, \psi, v, o.$
5.  $b, t, m, z, y, e, \gamma, n, s, p, \mu, \eta.$
6.  $c, b, a, t, m, \kappa, a, h, \beta, y, v, d, k, o.$
7.  $c, b, a, t, m, z, \kappa, a, h, \beta, y, v, d, x, o, l.$
8.  $b, a, t, m, z, \kappa, a, \lambda, i, \beta, y, v, \omega, d, o, g, Q.$
9.  $c, b, t, m, \kappa, a, \sigma, \beta, y, v, \omega, d, \epsilon.$
10.  $c, b, a, t, m, \kappa, a, \lambda, i, h, \sigma, \beta, y, v, \omega, d, C.$
11.  $c, b, a, t, m, z, \kappa, a, \lambda, i, h, \sigma, \beta, y, v, d, C, o, Q.$
12.  $c, b, a, m, z, \kappa, a, i, U, \beta, y, d, x, \epsilon.$
13.  $c, b, a, t, m, \kappa, a, i, h, \psi, U, \beta, \sigma, y, v, \epsilon, d.$
14.  $b, a, t, m, \lambda, i, W, y, q, d, \gamma, s, \rho.$
15.  $b, a, t, m, V, h, \psi, U, \beta, y, v, d, o.$
16.  $c, b, a, t, m, \kappa, a, i, h, \beta, y, v, o, d.$
17.  $b, m, z, H, \kappa, a, \beta, d, x.$

<sup>1</sup> H. S. Washington. Amer. Journ. of Science (3), 1887, 34, 281-287.

18.  $c, b, a, t, m, \kappa, a, h, \beta, y, v, \omega, d, k, o.$
19.  $b, t, m, z, \kappa, U, \beta, y, v, \omega, r, d, \epsilon.$
20.  $b, t, m, h, W, y, v, d.$
21.  $c, b, a, t, m, z, \kappa, a, \sigma, \beta, y, v, \omega, d, \epsilon.$
22.  $c, b, a, t, m, z, \kappa, a, h, U, \beta, y, v, \omega, d, x, o, \Theta, B, \epsilon.$
23.  $c, b, a, t, m, z, H, \kappa, a, \lambda, i, h, \sigma, \beta, y, v, \omega, d, k, o, \phi.$
24.  $b, a, t, m, a, h, \psi, y, v, d.$
25.  $c, b, a, t, m, \kappa, a, h, \psi, U, \beta, y, v, d, o.$
26.  $c, b, t, m, \kappa, a, i, h, \psi, U, \beta, y, v, \omega, r, d, \epsilon.$
27.  $c, a, t, m, \kappa, a, \lambda, i, h, \beta, y, v, d, x, o, \epsilon, \gamma.$
28.  $c, b, a, t, \kappa, a, h, \psi, \sigma, \beta, y, v, q, d, o.$
29.  $c, b, t, m, z, \kappa, a, \beta, y, v, q, d, \epsilon, Q.$
30.  $c, b, a, m, \kappa, a, i, h, \psi, y, v, q, d, o.$

*Projections.* — Figures 1–10, Plates 2 and 3, are clinographic projections to show the habits and some of the combinations. Figure 10 is a drawing of Crystal No. 5, which has the five new forms besides the two rare ones,  $e$  and  $\gamma$ . The new form  $(\bar{3}41)$  is a large face on this crystal. About two-thirds of the crystal is broken, otherwise it would probably have shown a rich combination of forms, as most of the common terminal forms are missing.

Figure 11, Plate 3, shows an orthographic projection on the base of all the forms, with their relative predominance.

Plate 1 exhibits a gnomonic projection of the forms. The pole of the projection, represented by the dark circle, is almost midway between the projection of the base  $(001)$  and the dome  $(\bar{1}01)$ , and since  $e'$  is almost  $\frac{p'_0}{2}$  the angles  $\phi$  and  $\rho$

for the faces on the positive side of the pole vary only a few minutes from those at corresponding distances from the pole on its negative side. The crystals of colemanite therefore have an apparent orthorhombic symmetry, and the definite orientation of some of the crystals had to be determined by the direction of extinction on the clinopinacoidal section, this direction making an angle of about  $6^\circ$  with the vertical axis, in the acute angle  $\beta$ .

The projection shows the excellent series of zones in which the forms lie. Most of the prominent zonal intersections on the negative side of the pole are occupied by faces, while

on the positive side, several of these prominent intersections are not represented by forms, as, for instance, the cross zone  $2p_0$  has only the one form (201) in it. Two of the negative pyramids,  $p(\bar{1}23)$  and  $w(\bar{1}82)$  have their projections on the positive side of the pole, and the latter form lies almost on the first meridian. The pyramidal forms which are new are represented by two circles, and the direction of the prism (310) and dome (801) by double-headed arrows. In this projection the prismatic forms necessarily have no points of intersection, but their directions are shown by the arrows.

#### METHOD

The two-circle goniometer, with which the measurements were made, has been fully described by Goldschmidt, together with detailed instruction in the calculation and gnomonic projection of the forms based on measurements with the instrument,<sup>1</sup> and also in a briefer way by Palache.<sup>2</sup>

*Advantages.* — This goniometer possesses manifest advantages over the ordinary reflection-goniometer, not alone in the simplicity and greater rapidity with which forms can be calculated from its measurements, but also because the two angular coördinates definitely locate the form, and when these angles have been recorded for every known form on crystals, as Goldschmidt has done in his "Winkeltabellen," a new form can readily be detected by comparing its angles with those recorded. It would be practically impossible to record all of the interfacial angles between known forms; consequently, as often happens, the measured interfacial angle is not given in the standard works on mineralogy, and quite frequently after long trigonometrical calculation the measured form turns out to be well known. The instrument, however, can also serve for the measurement of interfacial angles, and, in the opinion of the writer, is superior for this kind of measure-

<sup>1</sup> V. Goldschmidt. — Goniometer mit zwei Kreisen. *Zeitschrift für Krystallographie*, 1893. 21, 210-232. Die zweikreisige Goniometer (Modell 1896) und seine Justirung. *Idem* 1898, 29, 333-345.

<sup>2</sup> Charles Palache. — On Crystal Measurement by means of Angular Coordinates and on the Use of the Goniometer with two Circles. *American Journal of Science*, 1896 (4), 2, 298.

ment, as it really requires less time for the adjustment of the crystal.

One adjustment of a crystal in true polar position is often sufficient for the measurement of all the forms, and in the case of colemanite, less than an hour was required to measure the most complex combination.

*Polar Orientation.* — Every face of a crystal has its position defined with reference to a pole and a direction assumed as first meridian, when its two angular distances, respectively, from these are known. By means of the graduated vertical and horizontal circles of the instrument, these two angular coördinates,  $\phi$  and  $\rho$ , are readily derived. The plane, normal to the prismatic zone, is preferably chosen as the pole-face, and the great circle passing through the pole and the normal to the side pinacoid (010) as the first meridian. In systems with rectilinear axes the pole-face would then correspond to the basal-pinacoid, and the angles for the three pinacoids would be

	$\phi$	$\rho$
001	0°00	0°00
010	0 00	90 00
100	90 00	90 00

Since monoclinic crystals have no plane normal to the prismatic zone, the position of such a plane is best defined if prismatic faces are present. Colemanite possesses a well-developed prismatic zone, including both pinacoids, so the polar orientation of the crystals was readily accomplished. Each face of a crystal comes into reflection when it is normal to the line bisecting the angle between the telescope and collimator of the instrument, and this normal position must first be determined. Its angle on the horizontal circle  $H$ , is the  $h_0$  reading for all measurements. The method of finding  $h_0$  is quite simple. Having the telescope tightly clamped at a convenient distance from the collimator, a bright reflecting surface is then mounted approximately parallel to the vertical circle,  $V$ , and centred. It is then brought at the intersection of the crosshairs by turning  $H$  and  $V$ , and the reading on  $H$  taken, =  $h_1$ .  $V$  is then turned 180° and the reflection again brought into position by the adjustment tables and  $H$ . This reading on  $H$  =  $h_2$ . Then is  $h_0$  =

$\frac{1}{2}(h_1 + h_2)$ . This can be repeated until the reflection remains rigidly fixed at the intersection of the crosshairs, during a revolution of  $V$ . This final position of  $H$  is then the  $h_0$  and need never be changed.

The crystal of colemanite was mounted with its prismatic zone approximately normal to  $V$ , and  $H$  was clamped at  $90^\circ + h_0$ . The reflections from the prismatic faces were, by means of the centring and adjusting tables, brought to revolve directly in the line of the vertical crosshair, on turning  $V$ , and the crystal thus brought into true polar position, because a plane normal to the prismatic zone would then be at  $h_0$ . The reading on  $V$  for the clinopinacoid is the  $v_0$  for this circle, which would be different for each crystal. Owing to imperfect centring or other causes readings on  $V$  or on  $H$  for certain faces vary, when they should be the same; consequently both  $v_0$  and  $h_0$  can be corrected by averaging the different readings.

When the reflection of each face is brought at the intersection of the crosshairs, two readings, one on  $V = v$ , and one on  $H = h$ , are made; the face is then defined by two angular coördinates,  $\phi = v - v_0$  and  $\rho = h - h_0$ .

*Symbols p q.* — In place of the three indices of Miller for a terminal form, Goldschmidt uses the two indices  $p$  and  $q$ , and for calculations and in the gnomonic projection the two indices are preferable. The indices of Miller are readily transposed into those of Goldschmidt by making the last one equal to unity and not expressing it; thus 522 (Miller) becomes  $\frac{5}{2}1$  (Gdt). Furthermore 001 = 0, 100 =  $\infty$  0, 010 = 0  $\infty$ , 110 =  $\infty$ , 210 =  $2\infty$  =  $\frac{p}{q}\infty$ , 120 =  $\frac{q}{p}\infty$ . When  $p$  and  $q$  are equal, but one is expressed, thus 331 = 3. The zonal relations of forms are better shown by the two symbols, because all forms having the same  $p$ , or the same  $q$ , lie in the same straight line or zone; for example, it can be seen that the forms  $\frac{5}{2}2$ ,  $\frac{1}{2}2$ ,  $\frac{4}{2}2$  are tautozonal, whereas their Miller equivalents (564), (296), (8.15.10) do not show this relation so well.

*Gnomonic Projection.* — This projection shows the points of intersection of the face-normals, drawn from the centre of the crystal, upon a plane lying preferably normal to the prismatic zone and at a unit's distance from the centre of the

crystal. If  $h$  is the distance of this plane above the centre and is equal to the c-axis, and  $r_0$  is the length of the base normal, then in crystals with rectilinear axes  $r_0 = h = 1$ . In monoclinic crystals the base is oblique to the plane of projection, and with  $h = 1$ ,  $r_0$  is equal to  $\frac{1}{\sin \beta} = \frac{1}{\sin \mu}$ ; whence it follows that with  $r_0 = 1$ , then  $h = \sin \mu$ .

If with a radius of  $h = 1 =$  c-axis, a circle is described, then this circle would represent in ground plan a sphere of projection. The plane of projection would be tangent to this sphere at the end of the c-axis,  $S$  would be the pole of the projection, and  $SY$  the first meridian. Any face  $pq$  would have the point of intersection of its normal with this plane, located by the angle  $\phi$  which it makes with the first meridian  $SY$ , and the distance from the pole  $d' = \operatorname{tg} \rho$ . (Figure 1). The face  $pq$  is further defined by the two rectangular coördinates  $x' y'$ , whose values deduced from the right triangles

$$x' = \sin \phi \operatorname{tg} \rho$$

$$y' = \cos \phi \operatorname{tg} \rho$$

By means of the measured angles  $\phi$  and  $\rho$ , all forms can be plotted on cross section paper, and the coördinates  $x' y'$  give graphically the symbols for any form in terms of the  $x' y'$  of the unit form  $pq$ . In monoclinic crystals the projection of the base-normal lies in front of the pole at a distance  $e' = \operatorname{tg} \rho$ , and the distances  $p'_0$  and  $q'_0$  are the coördinates for the unit pyramid  $pq$  reckoned from the base; therefore for any values of  $pq$ ,  $x' = pp'_0 + e'$  and  $y' = qq'_0$ .

*Determination of  $e'$ ,  $\mu$ , and  $\beta$ .* — An average of twenty readings on the basal pinacoid gave  $\rho = 20^\circ 7'$ , with  $\phi = 90^\circ$ ;  $e' = \operatorname{tg} \rho = 0.3663$ . This value of  $e'$  was also obtained from the readings of the clinodomæ. For these forms  $x' = e'$ , and an average of forty-four values of  $x'$  for the domes 01 and 02 gave  $x' = 0.3663$ ; thus agreeing with the direct measurements. Also  $e' = \cot \mu = 69^\circ 53'$  and  $\beta = 180^\circ - \mu = 110^\circ 7'$ .

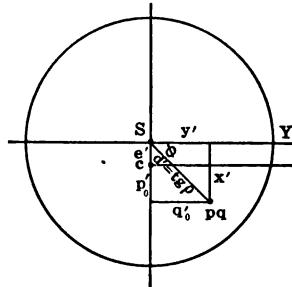


Fig. 1

*Determination of  $p'_0$  and  $q'_0$ .*—By means of the two formulae

$$x' = \sin \phi \operatorname{tg} \rho$$

$$y' = \cos \phi \operatorname{tg} \rho$$

the coördinates  $x' y'$  were calculated for all the best faces. For the positive forms  $pp'_0 = x' - e'$ , and for the negative  $pp'_0 = x' + e'$ . The symbols  $pq$  are simple multiples of the coördinates,  $p'_0q'_0$  of the unit form, therefore the values  $p'_0q'_0$  are readily deduced for each form. Taking fifteen of the best crystals as sufficient for the calculation of the elements, the averages of  $p'_0$  and  $q'_0$  were as follows:

$p'_0 = 0.7431$	12 meas.	$q'_0 = 0.5438$	12 meas.
7431	9 "	5426	9 "
7441	4 "	5435	9 "
7425	7 "	5426	7 "
7447	17 "	5426	17 "
7452	11 "	5435	11 "
7452	14 "	5432	14 "
7444	8 "	5429	8 "
7449	10 "	5417	10 "
7435	14 "	5424	9 "
7450	16 "	5436	16 "
7447	13 "	5423	13 "
7453	6 "	5437	6 "
7446	8 "	5425	8 "
7448	16 "	5431	16 "

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Average 0.7443	165 meas.	0.5430	165 meas.
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The elements for colemanite are therefore:

$$p'_0 = 0.7443; q'_0 = 0.5430; e' = 0.3663; \mu = 69^\circ 53'.$$

*Determination of the Polar Elements  $p_0$ ,  $q_0$  and  $e$ .*—The values  $p'_0$ ,  $q'_0$  and  $e'$  are the elements when  $h = 1$  and  $r_0 = \frac{1}{\sin \mu}$ ; therefore when  $r_0 = 1$ , these values must be multiplied by  $\sin \mu$ , to obtain the polar elements  $p_0$ ,  $q_0$  and  $e$ . Thus  $p_0 = p'_0 \sin \mu$ ,  $q_0 = q'_0 \sin \mu$  and  $e = e' \sin \mu = \cos \mu$ . The elements for colemanite then become

$$p_0 = 0.6989; q_0 = 0.5098; e = 0.3439.$$

*Determination of the Axial Lengths a and c.* — In systems with rectilinear axes  $q'_0 = \text{tg}(001: 011)$  and  $p'_0 = \text{tg}(001: 101)$ ; therefore with axis  $b = 1$ , the formulæ for such systems become  $c = q'_0$  and  $\dot{a} = \frac{c}{p'_0} = \frac{q'_0}{p'_0}$ .

In monoclinic crystals,  $c$  is also equal to the coördinate  $q'_0$ ; the clino-axis  $\dot{a}$ , however, is equal to  $\frac{\dot{a}}{\sin \mu}$ , therefore the formulæ for monoclinic crystals, in terms of  $p_0 q_0$  or  $p'_0 q'_0$ , when  $b = 1$ , become

$$c = q'_0 = \frac{q_0}{\sin \mu}; \dot{a} = \frac{q'_0}{p'_0 \sin \mu} = \frac{q'_0}{p_0} = \frac{q_0}{p_0 \sin \mu}$$

From these equations,  $\dot{a} = 0.7768$  and  $c = 0.5430$ ; giving the axial ratio for colemanite  $\dot{a}:b:c = 0.7768:1:0.5430$ .

*Record of the Measurements.* — As an illustration of the actual measurements and method of deriving the symbols for the forms, the work on Crystal 23 is given. Each crystal was sketched before measuring in order to show the relative size of the faces, and the faces were provisionally lettered. The kind of reflection was designated as good, fair, or poor (g. f. p.). Starting with the angle on the side pinacoid  $(010) = 139^\circ 44'$ , and plotting the  $v$ -angles in a circle in the direction of the hands of a watch, it is easily determined which forms are positive and which negative. The  $h_0$  for the instrument was  $57^\circ 54'$ , and the  $v_0$  for the crystal was  $139^\circ 44'$ .

From the columns  $pp'_0$  and  $qq'_0$  the symbols  $pq$  are at once apparent, since  $p'_0 = 0.7443$  and  $q'_0 = 0.5430$ . The record is simply a sample page of the note book, and it illustrates the simplicity and shortness of the work required to determine any form, as well as the neatness and compactness of the whole method.

Reflections.	Letter.					Prisms = $\operatorname{tg} \phi$			$pp' o$ $= x' - e'$ (positive)	$qq' o$	Sym- bols $p\ q$
		$v$	$h$	$\phi =$ $v - v_0$	$\rho =$ $h - h_0$	$\lg \sin \phi$	$\lg x'$ $\lg v'$	$\frac{z'}{v'}$			
g.	<i>b</i>	139°44	147°54	0°00	90°00				0		0 $\infty$
g.	<i>m</i>	193 40	147 54	53 56	90 00				$\infty$	1.3730	$\infty$
f.	<i>t</i>	209 27	147 54	69 43	90 00				$\infty$	2.7058	2 $\infty$
g.	<i>a</i>	229 44	147 54	90 00	90 00				$\infty$		$\infty$ 0
g.	<i>t'</i>	249 46	147 54	69 58	90 00				$\infty$	2.7400	2 $\infty$
g.	<i>m'</i>	265 44	147 54	54 00	90 00				$\infty$	1.3755	$\infty$
f.	<i>b'</i>	319 44	147 54	0 00	90 00				$\infty$		0 $\infty$
f.	<i>t''</i>	354 20	147 54	34 36	90 00				$\infty$	0.6898	$\infty$ 2
f.	<i>l</i>	345 07	147 54	25 23	90 00				$\infty$	0.4745	$\infty$ 3
g.	<i>m</i>	13 43	147 54	53 59	90 00				$\infty$	1.3752	$\infty$
g.	<i>x</i>	229 43	119 35	89 59	61 41	0	0.268556	0.268556	1.8559	1.4896	0 +2 0
g.	<i>c</i>	229 43	78 01	89 59	20 07	$\infty$	$\infty$	0			
g.	<i>c</i>	229 43	78 01	89 59	20 07	0	9.563811	9.563811	0.3663	0 0	0 0
g.	<i>p</i>	203 42	108 54	63 58	51 00	$\infty$	$\infty$	0			
g.	<i>p</i>	203 42	108 54	63 58	51 00	9.953537	0.045168	1.1096	0.7433	0.5420	+1 1
g.	<i>g</i>	217 55	127 12	78 11	69 18	9.642360	0.091631	9.733991	0.5420		
g.	<i>g'</i>	241 30	127 15	78 14	69 21	9.990697	0.422659	0.413356	2.5903	2.2240	0.5419 +3 1
g.	<i>g'</i>	241 30	127 15	78 14	69 21	9.311289	9.309474	9.733948	0.5419		
g.	<i>g'</i>	241 30	127 15	78 14	69 21	9.990777	0.423807	0.414584	2.5977	2.2314	0.5411 +3 1
f.	<i>h</i>	243 55	123 40	75 49	65 46	9.986555	9.309474	9.733281	0.5411	1.7876	0.5443 + $\frac{1}{2}$ 1
f.	<i>h</i>	243 55	123 40	75 49	65 46	9.389211	0.346674	0.333229	2.1539	9.735885	0.5443

Reflections.	Letter.	v	h	$\phi = v - v_0$	$\rho = h - h_0$	Prisms = $\operatorname{tg} \phi$			$pp'0$ $= x - e'$ (positive)	$qq'0$	Sym- bols $p\ q$
						$\operatorname{lg} \sin \phi$	$\operatorname{lg} z'$ $\operatorname{lg} y'$	$x'$ $y'$			
p.	i	261°51	129°39	57°53	71°45	9.927867 0.481814 9.725622	0.409681 0.207436	2.5685 1.6123	2.2022	1.6123	+3 3
g.	p'	255 45	108 57	63 59	51 03	9.953599 0.092406 9.642101	0.046005 9.734507	1.1117 0.5427	0.7454	0.5427	+1 1
g.	o	158 24	106 42	18 40	48 48	9.505234 0.055738 9.976532	9.560972 0.032270	0.3639 1.0771	0.0024	1.0771	0 2
g.	s	173 43	91 05	33 59	33 11	9.747374 9.815555 9.918659	9.562929 9.734214	0.3656 0.5423	0.0007	0.5423	0 1
g.	s'	285 44	91 06	34 00	33 12	9.747562 9.815831 9.918574	9.563393 9.734405	0.3659 0.5425	0.0004	0.5425	0 1
g.	o'	301 03	106 46	18 41	48 52	9.505608 0.058796 9.976489	9.564404 0.035285	0.3668 1.0846	0.0005	1.0846	0 2
p.	f	285 47	120 59	33 57	63 05	9.746999 0.294397 9.918830	0.041396 0.213227	1.1000 1.6339	0.7337	1.6339	+1 3
g.	v	120 28	106 50	19 16	48 56	9.518468 0.059817 9.974969	9.578285 0.034786	0.3787 1.0834	0.7450	1.0834	-1 2
g.	w	104 47	91 21	34 57	33 27	9.758050 9.819959 9.913630	9.578009 9.733589	0.3785 0.5415	0.7448	0.5415	-1 1
g.	n	49 45	78 39	90 01	20 45	0 9.578486 $\infty$	9.578486 $\infty$	0.3789 0	0.7452	0	-1 0

Reactions.	Letter.	v	h	$\phi = v - v_0$	$\rho = h - h_0$	Prisms = $\operatorname{tg} \phi$			$pp'_0 = x' - e'$ (positive)	$qq'_0 = x' + e'$ (negative)	Symbol
						$\operatorname{lg} \sin \phi$	$\operatorname{lg} x'$	$x'$			
g.	w'	354°41	91°24	34°57	33°30	9.758050 9.820783 9.913630	9.578833 9.734413	0.3792 0.5425	0.7455	0.5425	-1 1
g.	v'	339 00	106 53	19 16	48 59	9.518468 0.060582 9.974969	9.579050 0.035551	0.3794 1.0853	0.7457	1.0853	-1 2
g.	d	49 45	106 14	90 01	48 20	0.050647 ∞	0.050647 ∞	1.1237 0	1.4900	0	-2 0
g.	k	23 58	109 14	64 14	51 20	9.954518 0.096803 9.638197	0.051321 9.735000	1.1254 0.5433	1.4917	0.5433	-2 1
g.	u	5 44	115 18	46 00	57 24	9.856934 0.194141 9.841771	0.051075 0.035912	1.1247 1.0862	1.4910	1.0862	-2 2
g.	k'	75 30	109 11	64 14	51 17	9.954518 0.096027 9.638197	0.050545 9.734224	1.1234 0.5423	1.4897	0.5423	-2 1

*Calculated Table.* — Following is a calculation of the 47 forms to correspond to the tables arranged by Goldschmidt in his "Krystallographische Winkeltabellen." The calculations of colemanite given by the writer are based on the elements  $p_0 = 0.6989$ ;  $q_0 = 0.5098$ ; and  $e = 0.3439$ , and therefore show a slight difference in the angles from those given by him on page 100.

$a = .7768$	$lg a = 0.890309$	$lg a_0 = 0.155509$	$lg p_0 = 0.844415$	$a_0 = 1.4306$	$p_0 = .6989$
$c = .5430$	$lg c = 0.734800$	$lg b_0 = 0.265200$	$lg q_0 = 0.707463$	$b_0 = 1.8416$	$q_0 = .5098$
$\mu = \begin{cases} 69^\circ 53' \\ 180^\circ - \beta \end{cases}$	$lg h = \begin{cases} 9.972663 \\ lg \sin \mu \end{cases}$	$lg e = \begin{cases} 9.536474 \\ lg \cos \mu \end{cases}$	$lg \frac{p_0}{q_0} = 0.136952$	$h = 0.9390$	$e = .3439$

No.	Letter	Gdt.	Miller.	$\phi$	$\rho$	$\xi_0$	$\eta_0$	$\xi$	$\eta$	$\frac{x'}{(Prisms)} (x:y)$	$t'$	$d' = \tan \rho$
1	<i>c</i>	0	001	90°00'	20°07'	20°07'	0°00'	20°07'	0°00'	0.3663	0	.3663
2	<i>b</i>	$0\infty$	010	0°00'	90°00'	0°00'	90°00'	0°00'	90°00'	0	$\infty$	$\infty$
3	<i>a</i>	$\infty 0$	100	90°00'	90°00'	90°00'	0°00'	90°00'	0°00'	$\infty$	0	"
4	<i>l</i>	$3\infty$	310	76°20'	"	"	90°00'	76°20'	13°40'	4.1227	$\infty$	"
5	<i>t</i>	$2\infty$	210	69°58'	"	"	"	69°58'	20°02'	2.7419	"	"
6	<i>m</i>	$\infty$	110	53°53'	"	"	"	53°53'	36°06'	1.3709	"	"
7	<i>z</i>	$\infty 2$	120	34°26'	"	"	"	34°26'	55°34'	0.6854	"	"
8	<i>H</i>	$\infty 3$	130	24°33'	"	"	"	24°33'	65°26'	0.4570	"	"
9	<i>k</i>	01	011	34°00'	33°13'	20°07'	28°30'	17°50'	27°01'	0.3663	0.5430	0.6550
10	<i>a</i>	02	021	18°38'	48°54'	"	47°22'	13°56'	45°34'	"	1.0860	1.1461
11	<i>V</i>	$+10$	101	90°00'	47°59'	48°00'	0°00'	47°59'	0°00'	1.1106	0	1.1106
12	<i>λ</i>	$+20$	201	"	61°40'	61°40'	"	61°40'	"	1.8549	"	1.8549
13	<i>ρ</i>	$+30$	301	"	68°57'	68°57'	"	68°57'	"	2.5992	"	2.5992
14	<i>i</i>	$-10$	101	90°00'	20°42'	20°42'	"	20°42'	"	0.3781	"	0.3781
15	<i>h</i>	$-20$	201	"	48°18'	48°18'	"	48°18'	"	1.1223	"	1.1223
16	<i>g</i>	$-10$	502	"	56°13'	56°13'	"	56°13'	"	1.4944	"	1.4944
17	<i>W</i>	$-30$	301	"	61°49'	61°49'	"	61°49'	"	1.8666	"	1.8666
18	<i>ψ</i>	$-40$	401	"	69°02'	69°02'	"	69°02'	"	2.6109	"	2.6109
19	<i>U</i>	$-60$	601	"	76°17'	76°17'	"	76°17'	"	4.0996	"	4.0996
20	<i>f</i>	$-80$	801	"	79°51'	79°51'	"	79°51'	"	5.5882	"	5.5882
21	<i>σ</i>	$+3$	331	57°55'	71°56'	68°57'	58°27'	53°40'	30°19'	2.5992	1.6290	3.0675
22	<i>β</i>	$+1$	111	63°56'	51°02'	48°00'	28°30'	44°18'	19°58'	1.1106	0.5430	1.2362
23	<i>y</i>	$-1$	111	34°51'	33°29'	20°42'	"	18°23'	26°55'	0.3781	"	0.6617
24	<i>v</i>	$-2$	221	45°56'	57°22'	48°18'	47°22'	37°14'	35°51'	1.1223	1.0860	1.5617
25	<i>q</i>	$-3$	331	48°53'	68°01'	61°49'	58°27'	44°19'	37°34'	1.8666	1.6290	2.4776
26	<i>n</i>	$+14$	141	27°05'	67°42'	48°00'	65°17'	24°54'	55°28'	1.1106	2.1720	2.4396
27	<i>ω</i>	$+13$	131	34°17'	63°06'	"	58°27'	30°09'	47°28'	"	1.6290	1.9716
28	<i>e</i>	$+12$	121	45°38'	57°13'	48°00'	47°22'	36°57'	36°00'	1.1106	1.0860	1.5533
29	<i>η</i>	$+1\frac{1}{2}$	232	53°44'	54°01'	"	39°10'	40°44'	28°35'	"	0.8145	1.3772
30	<i>r</i>	$-1\frac{1}{2}$	232	24°54'	41°55'	20°42'	"	16°20'	37°18'	0.3781	1.1100	0.8980
31	<i>d</i>	$-12$	121	19°11'	48°59'	"	47°22'	14°22'	45°27'	"	1.0860	1.1500
32	<i>x</i>	$-13$	131	13°04'	59°07'	"	58°27'	11°11'	56°43'	"	1.6290	1.6724
33	<i>C</i>	$+10.1$	10.1.1	86°01'	82°43'	82°42'	28°30'	81°42'	3°57'	7.8093	0.5430	7.8281
34	<i>k</i>	$+31$	311	78°12'	69°22'	68°57'	"	66°21'	11°02'	2.5992	"	2.6553
35	<i>φ</i>	$+41$	522	76°18'	66°26'	65°49'	"	62°56'	12°32'	2.2270	"	2.2922
36	<i>o</i>	$-21$	211	64°11'	51°16'	48°18'	"	44°36'	19°52'	1.1223	"	1.2468
37	<i>Θ</i>	$-31$	311	73°47'	62°47'	61°49'	"	58°38'	14°23'	1.8666	"	1.9441
38	<i>B</i>	$-41$	411	78°15'	69°26'	69°02'	"	66°27'	10°59'	2.6109	"	2.6663
39	<i>ε</i>	$-23$	231	34°34'	63°11'	48°18'	57°27'	30°25'	47°18'	1.1223	1.6290	1.9782

$a = .7768$	$\lg a = 9.890309$	$\lg a_0 = 0.155509$	$\lg p_0 = 9.844415$	$a_0 = 1.4306$	$p_0 = .6989$							
$c = .5430$	$\lg c = 9.734800$	$\lg b_0 = 0.265200$	$\lg q_0 = 9.707463$	$b_0 = 1.8416$	$q_0 = .5098$							
$\mu = \begin{cases} 69^\circ 53' \\ 180^\circ - \beta \end{cases}$	$\lg h = \begin{cases} 9.972663 \\ \lg \sin \mu \end{cases}$	$\lg e = \begin{cases} 9.536474 \\ \lg \cos \mu \end{cases}$	$\lg p_0 = 0.136952$	$h = 0.9390$	$e = .3439$							
No.	Letter.	Gdt.	Miller.	$\phi$	$\rho$	$\xi_0$	$\eta_0$	$\xi$	$\eta$	$\frac{x'}{(Prisms)} (x : y)$	$y'$	$\frac{a'}{= \lg \rho}$
40	$Q$	- 24	241	27°19'	67°45'	48°18'	65°17'	25°08'	55°19'	1.1223	2.1720	2.4450
41	$s$	- 34	341	40°40'	70°45'	61°49'	"	37°58'	45°43'	1.8666	"	2.8640
42	$\gamma$	- 32	321	59°48'	65°09'	"	47°22'	51°39'	27°09'	"	1.0860	2.1598
43	$w$	- $\frac{1}{2}4$	182	0°09'	65°17'	0°20'	65°17'	0°08'	65°17'	0.0058	2.1720	2.1715
44	$p$	+ $\frac{1}{2}2$	142	34°12'	52°43'	36°26'	47°22'	26°34'	41°08'	0.7384	1.0860	1.3133
45	$u$	+ $\frac{1}{2} \frac{1}{2}$	164	34°08'	44°32'	28°55'	39°10'	23°11'	35°29'	0.5523	0.8145	0.9841
46	$\mu$	+ $\frac{1}{2} \frac{3}{2}$	165	38°19'	39°43'	27°15'	33°05'	23°20'	30°05'	0.5151	0.6516	0.8306
47	$P$	- $\frac{1}{2} \frac{1}{2}$	123	18°05'	20°51'	6°44'	19°54'	6°20'	19°46'	0.1182	0.3620	0.3808

It is to be regretted that up to the present no descriptive articles dealing with crystals of the tetragonal and triclinic systems according to the two-circle method have yet appeared in the English language. Very good illustrations of the application of this method to crystals of these systems have been published in German, viz.:

#### TETRAGONAL SYSTEM

Über Kalomel von Victor Goldschmidt und B. Mauritz (Zeitschrift für Krystallographie, Band XLIV).

#### TRICLINIC SYSTEM

Krystallberechnung im triklinen System illustriert am Anorthit von L. Borgström und Victor Goldschmidt (Zeitschrift für Krystallographie, Band XLI).

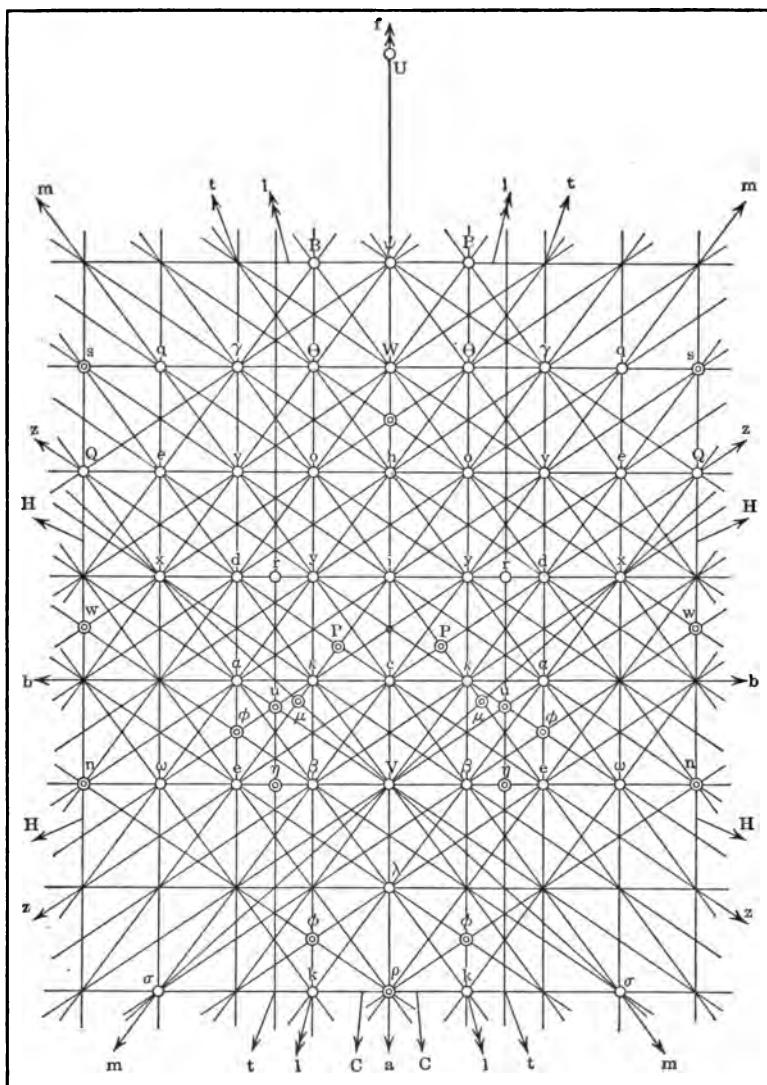
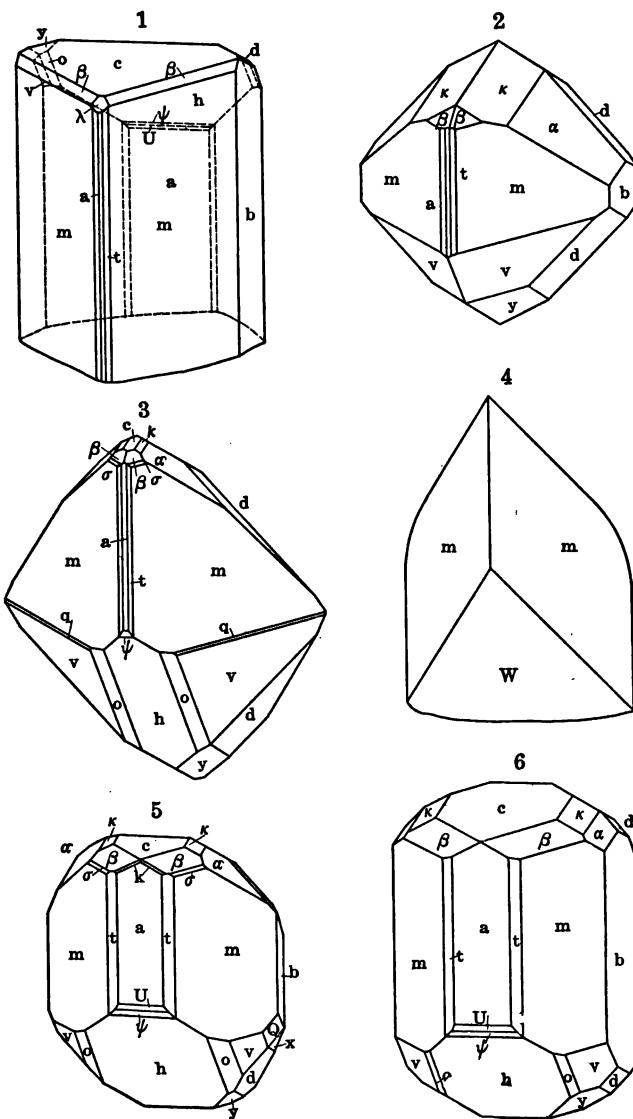
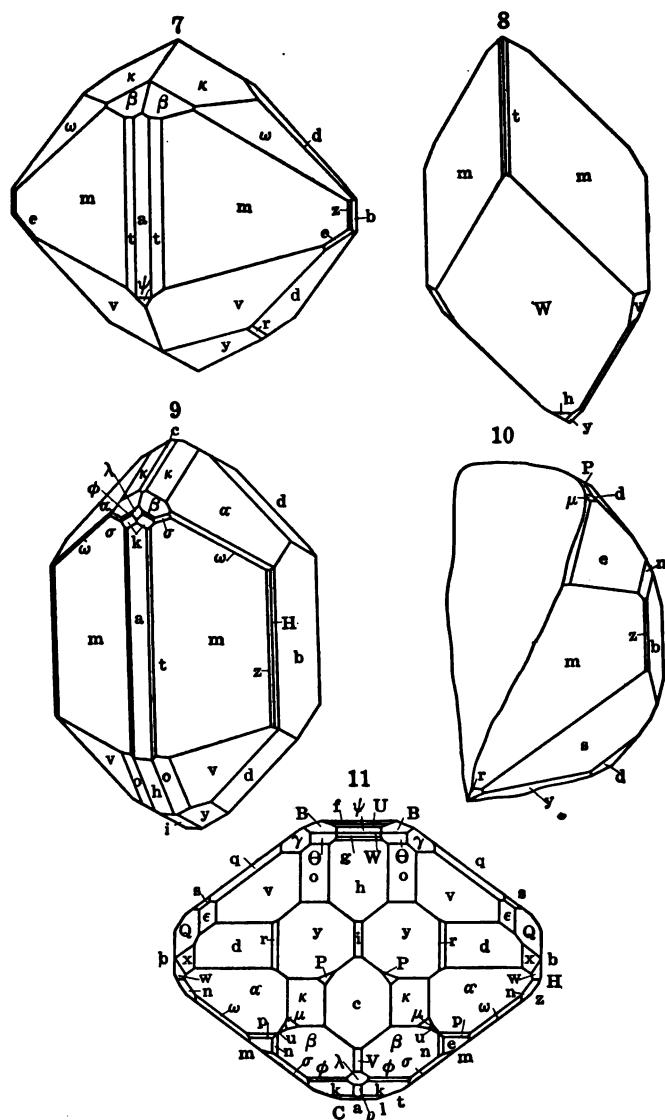


Plate I



Colemanite

Plate 2



## **Colemanite**

### Plate 3



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